Techniques Grading: Mastery Grading for Proofs Courses

ABSTRACT. Mastery grading is an approach to grading in which students are assigned term grades based on whether they meet certain enumerated objectives, rather than accumulating points. In this note, I describe my experiences using a mastery system, which I call techniques grading, which applies the insights behind the standards-based and specifications grading flavors of mastery grading to a proof-intensive course. In techniques grading, each objective assesses, in a binary (pass/fail) way, whether a student has mastered a specific proof technique.

I describe my implementation of this system in a transitions course and an undergraduate real analysis course. I discuss how the system works: how I developed the grading objectives, how individual assignments are assessed, and the collation of each student's work into a final portfolio. I provide a theoretical assessment of techniques grading within the mastery grading framework, and some evidence from student surveys and my own impressions that the system meets its goals: improving the quality of student work; increasing student satisfaction; reducing grade-grubbing; and instilling mindfulness, good work practices, and pride.

1. Introduction: Bowman's dilemma

The framework I present here is what I call *techniques grading* (TG), a system of mastery grading adapted for proofs-based courses, where the objectives are framed as the *techniques* of the subject of the course.

TG owes much to both specifications grading (SG) and standards-based grading (SBG). To many in the mastery grading community, this lineage may seem oxymoronic. Bowman succinctly and eloquently describes the difference between specifications grading (SG) and standards-based grading (SBG):

> Standards emphasize content.
> Specifications emphasize activity. [3]

Bowman's distinction between *content* and *activity* is apt. It presents a (or maybe just reiterates an ancient) dilemma: we "really care" about content, but as we are reminded in generating student learning objectives, our goals for a course ought to be definite and measurable—especially as presented in student-facing materials such as course policies and syllabus. Which system ought we choose: standards, or specifications? Do we care more about *content*, or more about *activity*?

I propose to square the circle of Bowman's dilemma by making a strong philosophical claim: what is at stake in almost all proofs courses is not facts about the mathematical objects in play, but rather the habits of mind and tricks of the trade of combinatorics or algebra or analysis. To successfully impart this "yoga of thinking" of the subject we are teaching [7] means that our students will be able to think critically not only generally, but also in the particular way that experts in the domain do. On this view, precise statements of theorems and definitions, and indeed the ability to find proofs for "tricky" exercises, are important mainly insofar as they serve the broader goal of learning how to think like an analyst, or an algebraist, or a combinatorialist. TG is an attempt to answer the question: how can we incentivize "thinking like an analyst" or "thinking like a topologist"?

With this in mind, TG is SBG from ten thousand feet: we identify a relatively small number of common proof techniques used in the subject and content themes which run

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through it. On the other hand, TG runs on the hardware of SG: its objectives are stated in binary, product-focused terms.

The present discussion is based on my experience over implementing TG in my courses at North Carolina State University, a large comprehensive public research university. The description given here is the result of tweaks and refinements over the course of eight semesters of a freshman/sophomore-level transitions course and one semester of a junior/senior-level real analysis course. Pedagogically, these are relatively standard lecture-and-discussion based courses.

The remainder of this article is organized as follows. In §2, I lay out the motivating principles and goals of TG, explaining why TG is a distinct enough form of mastery grading to merit its own label and how it serves these goals. In §3, I give a brief description of the system with some examples of objectives (complete lists of objectives for a transitions course and a real analysis course can be found in Appendices A and B respectively). In §4, I evaluate TG against theoretical criteria proposed by Nilson [8] and others, and explore how its various moving parts contribute to meeting them. In §5, I give some observational and anecdotal evidence of the success of TG in my courses. §6 is a user's guide to selecting objectives and implementing TG in your course. §7 offers some concluding reflections.

2. Motivating principles and pedagogical goals

TG is an attempt to implement principles from several flavors of mastery grading in a course whose primary curricular objective is getting students to write correct mathematical proofs, for example algebra or analysis. I will use the following labels for ease of reference:

binarity: Objectives are *binary* or pass/fail.
product focus: Objectives are stated in terms of the student's work product.
multiple chances: Students are given many chances to achieve each objective.
transparency: The system is clear to students.

Binarity, product focus, and transparency are the distinctives of SG [8].

Nilson's examples of specification systems come from engineering courses and general education courses with one or several major assignments. What these systems have in common is a focus on the deliverable outputs, such as lab reports, white papers, etc., for which binarity makes sense: the student either did or did not complete the work. One way that Nilson's principles have been implemented in mathematics courses is to set some kind of threshold performance standards, such as [11] and [9], where the required objectives for an "A" letter grade are: *either pass all 3 Exams, pass all 6 Quizzes, pass all but 2 Worksheets, and earn at least 80% on the Final; or earn 90%-100% on the final.* Such a system is certainly simple, and it's certainly transparent as to the activities that will occur in the course. Setting a numerical threshold for performance seems less binary and more akin, in some ways, to SBG. Furthermore, it is opaque as to the relationship between the grade and the content of the course: what's on the exam?

The other, standards-based, horn of Bowman's dilemma, is the importance we place (and of right, ought to place) on tying grades to the course content.A truly comprehensive list of possible content objectives (e.g. [6], which it should be noted was not developed for mastery grading) may run into the hundreds of items; so in order to develop a workable scheme, the instructor more or less has to pick and choose among objectives, or adopt a "at least n from among. . ." strategy for the objectives.

TG takes Nilson's transparency criterion one step further: to use the syllabus as a means of communicating the big picture of the subject (in addition to communicating the details of course administration). By selecting appropriately broad techniques and themes, we ensure the multiple attempts which are a foundation of all mastery grading systems—a properly chosen technique will be applicable to a broad range of problems, giving the student many chances to demonstrate mastery of it. Moreover, the instructor's choice of techniques and themes becomes an important device for communicating the broad structure of the subject to the student.

3. Techniques Grading

The TG objectives come in three categories: *technique* objectives, which constitute the bulk of the objectives, *exam* objectives, which concern performance on quizzes and exams, and *participation* objectives. To achieve a letter grade, students must meet the objectives for that letter grade in each category; that is to say, a student's final grade is the minimum of the letter grades in each category. Students who fail to meet the "C" objectives in any one category receive a "D" or an "F". In case of disagreement between categories, I may give a plus or minus grade. Both pluses and minuses and the distinction between "D" and "F" are judgment calls influenced primarily by the technique objectives.

3.1. Technique objectives. Each technique objective is achieved by writing a clear, correct, and complete proof which uses the referenced technique. Proofs are assigned as weekly homework sets.

In my transitions course (topics: logic, sets, functions, relations, and cardinality), the techniques are mainly proof techniques. To receive each possible letter grade, a student must complete all technique objectives at that letter-grade and below, so that to earn an "A" a student must complete all of the "A", all of the "B", and all of the "C" technique objectives. The full list of technique objectives can be found in Appendix A; some representative examples are:

- **A::** use the Well-Ordering Principle; show that a predicate P is true for every finite set X
- **B::** use complete induction
- **C::** a proofs by contradiction; a proof establishing cardinality of a set

In my real analysis course, technique objectives are sorted into two levels (Level II techniques are generally trickier than Level I). The full list of technique objectives can be found in Appendix B; some representative examples are:

- **Level II::** use the sequential characterization of limit; compute a limit superior or limit inferior; use topological compactness; establish topological compactness; integrate by showing a 'bad' set is small
- **Level I::** use the sequential characterization of continuity; use sequential compactness; establish sequential compactness; use the Cauchy Criterion for integrability; use the ϵ-δ definition of limit

Each grade requires a certain number of proofs, demonstrating certain number of techniques in each tier:

- **A::** 80 proofs; 30 techniques, including 15 level-II techniques; compute three integrals
- **B::** 65 proofs; 20 techniques, including 10 level-II techniques; compute two integrals
- **C::** 50 proofs; 10 techniques; compute an integral

Students may demonstrate multiple techniques in a single proof; for example a proof that *The union of finitely many compact sets is compact* may involve both using and establishing either topological or sequential compactness.

Quite clearly, neither of these lists of objectives is entirely comprehensive in the sense of learning objectives. There is much else that could and should be included in a compendium of what students should know. But the techniques are intended to stress the major themes and tricks of thinking.

The requirement that students complete a certain number of proofs throughout the semester is in practice redundant—students who meet the objectives for "A" all meet this quantity along the way—but the requirement gives students some sense of chipping away at their objectives, which I find helpful in motivating students.

3.2. Grading of proofs. The technique objectives themselves are binary: a student either does or does not submit a qualifying proof. The way proofs are graded is also pass/fail, with a modification due to Talbert [10]. Each proof receives a mark of *satisfactory* (S), *progressing* (P), or *unsatisfactory* (U). Only S proofs count, and the criterion for receiving a S is quite high: the proof must meet the formal requirements (stating the claim, indicating where the proof starts and ends, using correct notation) and successfully prove what it sets out to prove. A P proof has some flaws (minor or major) which are correctable. A U proof either has catastrophic mistakes, or uses a doomed strategy, or has other essentially uncorrectable issues.

Students are allowed to correct and resubmit P proofs. Such resubmissions are graded on the same basis as initial submissions: a resubmitted proof could again receive a mark of P, in which case the student can correct and resubmit again. There is no limit to the number of resubmissions.

It's important to note that resubmission opportunities are given on the basis of whether the submitted attempt is fixable, not on my estimation of whether the student is able to successfully complete a proof of the assigned claim. Thus resubmissions must be revisions of the submitted proof, rather than an entirely new attempt.

3.3. Exam objectives. I reserve exams (hence, reserve exam objectives) for multiple-choice, short-answer, or essay questions which it makes sense to grade on a points basis. In the transitions course, for example, I may have students read a proof and identify which claim it is the proof of; in the analysis course they should recall statements of theorems and definitions, and answer short essay questions like *Explain the mnemonic* $MST + MSST = BW$.[1] The standards for exam performance are 'low' compared with most points-average systems:

- **A::** 70% and 85% on each of two exams; 80% on the final
- **B::** 50% and 70% on each of two exams; 70% on the final
- **C::** 50% on each of two exams (including the final)

"Exams" here includes two midterm exams and a cumulative final exam; thus a student may completely botch one midterm exam and still achieve a grade of "A".

3.4. Participation objectives. Participation objectives are those which require students to complete ungraded assignments. These include completing a certain percentage of reflection journal entries, completing a creative assignment such as writing their own analogy for induction [5], and doing preclass reading and viewing.

3.5. The portfolio. I assess students' technique objectives by means of an end-of-term portfolio. In this portfolio, students collate only S-marked proofs. Each proof must be accompanied with a sentence or two explaining which standards it meets, and indicating why. For example, a student might write "I used the Sequential Characterization of Continuity to show. . ." Portfolios are due before the final exam; I check them to make sure the claimed standards have actually been met. Typically each student has a standard or two they have

[1] The answer is: the Bolzano-Weierstraß Theorem can be proved by combining the Monotone Sequence Theorem and the Monotone Subsequence Theorem.

misidentified; that is, the student submits a valid S proof but it doesn't meet that standard. The portfolio audit gives such student another chance to submit a proof which does meet the standard in question. Almost always, the student has such a proof at hand.

4. How does TG meet its goals?

My motivation for chosing any form of mastery grading was to ensure and encourage students to exhibit *mastery* of the material. On this ground, my implementation of TG succeeds on two levels: at the per-proof level, to receive a mark of S, a student must have used the technique at quite a high level. They must also, in the final portfolio, recognize and explain that they have done so. In order to receive credit, then, a student must not only be able to use a technique, but they must demonstrate thoughtfulness about its use.

At the broader level, the process of compiling the final portfolio gives students occasion to reflect on the course as a whole and an opportunity to see the themes running through the material. That is to say, TG encourages mastery of the course as a whole. This mastery also includes some big-picture aspects that are difficult to otherwise successfully convey. By way of example, a common trick in analysis is to clarify a problem by converting a metric or topological statement into a statement about sequences. Students are often mystified by this. In working to complete the technique objectives, and in the compilation of the final portfolio, students are exposed to the principle that *everything in metric topology can be done with sequences*.[2] What I formerly struggled to convince students of by way of commentary is woven into the structure of the course itself.

4.1. The motivating principles. TG meets all four of its stated aims: binarity, product-focus, multiple chances, and transparency. Binarity and product-focus are clear from the structure of the system.

At the per-proof level, the S/P/U marking scheme satisfies the *multiple-attempts* criterion. TG gives multiple attempts on a broader level, too. Because the techniques are genuinely common ones, for any given technique objective there are many relevant assigned proofs on which to demonstrate it. Typically I assign between 100 and 200 proofs in a semester, depending on the course; in principle a student could meet all the technique objectives for an "A" with around 30 of these proofs.

As for *transparency*, I hope a glance at the sample lists of technique objectives will convince the reader that the grading system itself communicates not only what the student is expected to do, but indeed what the entire course is about.

4.2. Nilson's criteria. Nilson [8] lists 15 criteria which she argues a grading system ought to satisfy. Here's how I would assess TG on these criteria:

Uphold high academic standards: Precisely to the extent the bar for an S proof is set high enough, TG encourages rigor. As in SG, it is not possible to pass on accumulated partial credit.

TG allows the instructor to assign meatier proofs (for example, proofs of named theorems) in the analysis course: students need an opportunity to pick which techniques to use, so I have to let them have more open-ended opportunities (see Appendix B). S/P/U marking makes assigning intense problems feasible without failing students.

[2] The precise mathematical statement is: a metrizable topological space is uniquely determined by the convergence of its sequences. Such a statement is, of course, beyond the scope of a first-semester real analysis course.

Reflect student learning outcomes: This is apparent—both to the reader here, but also importantly to the students—from the construction of the techniques list.

Motivate students to learn: The collation of the end of term portfolio encourages students to put together work they will be proud of. The assignment of more in-depth problems makes the course more compelling to students.

Motivate students to excel: Our transitions course serves statistics and mathematics education majors as well as mathematics majors. Some of these students come in with the express goal of doing the minimum to pass the class. TG treats such students with more respect, because the "C" student has still produced high-quality, worthy work. Under TG, a "C" is not simply a failed attempt at an "A".

Reduce student stress: Exams are stressful for students. In math major courses, this often manifests as a concern that the student will not be able to come up with a novel proof in a 60- or 90-minute period. TG exams eschew timed proof-writing, focusing instead on items where a numerical score makes natural sense.

Discourage cheating: A major predictor of student cheating on an exam is the would-be cheater's anxiety about the exam [2]. Reducing stress levels helps students think more rationally and make the right decision in the end.

Make students feel responsible for their grades: In the collation of the final portfolio, it is the student's responsibility to identify and explain how each proof meets the technique objectives it is submitted for.

Minimize conflict between faculty and students: I have dealt with essentially no grade-grubbing or points-begging since implementing TG; my prior experiences with both the transitions course and the analysis course were characterized by it. Students do bring questions about P-graded proofs to my office hours; but the discussion is focused on how to improve the work, rather than being an argument about what various mistakes are "worth".

Save faculty time: S/P/U marking is a godsend in terms of time. Hopeless U proofs are quickly identified and require no further grading (though I usually mark them up for the student's benefit). The difference between P and S, while a judgment call, takes much less time than awarding partial credit.

By placing the responsibility for collating the final portfolio in the students' hands, I save myself the time associated with recording anything other than the number of S proofs achieved, exam scores, and participation in other assignments.

Give students feedback they will use: The S/P/U system was essentially designed with this criterion in mind. If they do not attend to feedback on a P proof, students will receive no credit for that proof.

Make expectations clear (11) and be simple (15): TG takes a bit of work to implement, and it takes students some time to adjust to it; but at the end of the day, all of the expectations are written out on the first day of the semester. In my experience, the greatest difficulty students have with TG is accepting its simplicity. Most students' questions about the mechanics of TG are along the lines *Do you really mean this? Where's the hidden catch? Where's the fine print?*. The system is too simple to be believed.

Foster higher-order cognitive development and creativity: The revision process, which is central in both higher-order cognition and creative thinking, is where most students spend most of their time in a TG course.

Assess authentically: The standard for a S proof is an authentic one: is the proof complete and correct? Fundamentally this is the same sort of evaluation employed by the referee of a research journal.[3]

Have high interrater agreement: While there is some judgment required by S/P/U marking, most mathematics instructors can apply the "I know it when I see it" test [1] to identify complete, correct proofs with high interrater agreement.

5. Evaluation in practice

As set out in §4, TG should on theoretical grounds meet the goals described in §2. I have collected some evidence that it does—and has further pedagogical benefits—from student surveys. In addition to my own informal impressions of these courses and personal interactions with students, the quotes and anecdotes presented here come from several sources:

- anonymous student evaluations of teaching
- blinded student surveys associated with a critical and creative thinking curriculum [4] in the transitions course
- nonblinded responses in students' reflection journals

5.1. Students' learning about mathematics. The proportion of students who genuinely understand the "big picture" of the course certainly feels much higher under TG than it ever did on a points-average system. Anecdotally speaking, the writing quality and mathematical sophistication of student work in my TG courses has improved over my previous experiences with points-average systems. The most dramatic improvement is among students who ultimately receive a course grade of "C"; whereas previously a grade of "C" would have indicated consistent, but consistently mediocre, student work, under TG these students submit complete, correct proofs showing a mastery of a significant portion of the course material.

As noted above, a significant number of students in the transitions course come in with the ambition to achieve a "C" only. Several have reported to me they feel more secure with TG than under a points-average system—it's clear what they need to do to get the "C". In fact a number of these students find that the satisfaction of successfully revising a P proof into an S proof is invigorating, so much that they go on to achieve a "B" or an "A". Giving students the freedom to choose their level of investment seems to do a better job of encouraging such students than does mandating a high level of investment.

5.2. Students' learning about learning. We all know, as expert learners ourselves, that failure is an important and necessary part of learning. Part of my decision to adopt any kind of mastery grading was that I wanted my grading scheme to incentivize learning through failure—and to transparently normalize failure. One of the major lessons in a transitions course is how to deal with the frustration of a knotty problem, or a problem that is knottier than it seemed. To my delight, the grading system itself has transmitted this message better than any preaching or counseling on my part ever could. I asked students to give advice to future students in the course; a common theme was the value of failure in the course:

- "failing is a part of learning in this course"
- "Do not be afraid of making mistakes."

[3] If anything, research-journal mathematics is frequently held to a lower standard. I tell my students this, and they take pride in it.

- "learn that mistakes are okay, and that they make you better."

A related theme that emerges from the student surveys is that students actually seem to appreciate red ink, that is, critical comments they receive on their non-S proofs:

- "The amount of U's and P's I got severely outnumbered my S proofs, but I learned from them."
- "We were encouraged to learn from what we did wrong in a positive way!"
- "the hardest grad[ing] I have ever had in any class, but if [it] wasn't I wouldn't understand the material the magnitude that I do"

During the term, students seem to pay more attention to the comments their proofs receive. For P-marked proofs, it's obvious why: they are invested in making the revisions needed to get an S. But I have found students also read—and ask about in office hours—the comments I leave on their S and U proofs. (This is in contrast to points-average marking, where students feel no need to take feedback on either high- or low-marked problems.)

5.3. Student motivation. Anecdotally, TG does a good job of encouraging students throughout the course, while still requiring high levels of absolute performance. S/P/U marking "provides rewards for learning over time, rather than for getting it right on the first try," as one student wrote.

The end-of-term portfolio also motivates by helping students take pride in their work. A number of students have requested to rewrite their portfolio proofs, even though their original submissions would have met the standards—they want to make the whole package presentable. Almost all of my students want to keep their portfolios after I have marked them.

6. Adapting TG

To use the TG system in any proofs-based course should be fairly straightforward. The non-content objectives (participation, exam performance, etc.) are just the activities you expect students to do as a matter of course. I have found that almost all students meet the A-level participation standards (90% of preclass activities and reflections) anyway; these objectives mainly exist to send a message, rather than being used for actual assessment of or discrimination among students.

The difficult work is determining the technique objectives. As mentioned above, the key is to think in terms of techniques and themes, rather than objects or problems. You also need to think of multiple chances—each objective has to be something that comes up frequently.

My method for selecting technique objectives is as follows. First, start from a collection of the sorts of proofs you would normally assign (for example, the homework assignments for the past three semesters) and make a tally of how often each technique occurs. Next do the same with the proofs in the textbook you plan to use for the course. Finally, round out the techniques by referring to a reasonably complete list of learning objectives (perhaps one you've created for the course or a publicly-available one, such as [6]).

The named theorems of your course have names because they are so often invoked. Thus they are natural candidates for technique objectives ("Use Taylor's Theorem.") Any given pair of equivalent definitions probably makes for one or more good technique objectives. For example, the Heine-Borel Theorem in analysis says that (in the real context) sequential compactness, the Heine-Borel property, and topological compactness are all equivalent; this furnishes six of the technique objectives for my analysis course.

The resulting list of techniques is likely too long to be used directly; weed out any that occur only two or three times in homework or the textbook. Removing something as an objective is not the same as removing it from the course entirely—there is room for that content in direct instruction, assigned reading, and inclusion in homework problems that any particular student might not ultimately master. Indeed I typically use this "leftover mathematics" to generate problems to assign my students.

7. Reflections and Conclusion

Though I have found TG to be successful—indeed, transformational—for the courses I've used it in, I have encountered some roadbumps.

7.1. Student buy-in. As with any form of mastery grading, one major obstacle in implementing TG is getting student buy-in. At first glance, the syllabus is much more complicated than a syllabus for a points-average system; this causes some initial disquiet among students. I have found most success addressing these matters head-on: I am open with students that this grading system is different from what they're used to. I point out its goals and the fact that the syllabus is lengthy because it lists in some detail the skills students will acquire in the course. I also point out to students that the TG system prevents the strategy of banking points early on in the semester; some students do consider this a bug of the system, at least initially. In the end, student surveys show that, by the end of the semester, nearly all students come around to the view that they learn no worse under TG than under a traditional grading scheme; the majority of comments mention it positively.

7.2. Problem cases. About one student per semester presents what I call a *problem case* for TG: that is, my subjective estimation of the grade the student should receive differs from their grade according to TG. Sometimes this indicates that a particular technique objective needs to be tweaked; sometimes, in reviewing the student's record of submitted work, I find the TG grade is justified. However, this phenomenon occurs in most grading systems, including points-average systems. At least with TG, I have the opportunity to revise the system for future semesters.

Another way to put this is: I am not certain I have got the entire scheme right—in fact I am certain there are many tweaks left. But experience in the transition course shows each iteration improves course outcomes significantly, and also yields new insights about what's really important in the course.

But one thing is certain: for me, there is no going back to points-average grading in either course.

Appendices

Appendix A. Technique Objectives for a Transitions Course

The following table presents the technique objectives for my transitions course. Throughout the semester, some proofs are marked $\star$ or $\star\star$ according to their difficulty.

A	proof that something is unique compute the equivalence classes of an equivalence relation use the Well-Ordering Principle show that P is true for every finite . . . 5 $\star$ proofs and 5 $\star\star$ proofs
B	a proof by complete induction proof that two definitions are equivalent proof of a calculus fact proof that a relation is an equivalence relation 5 $\star$ or $\star\star$ proofs
C	proof by ordinary induction proof by contradiction an open-ended problem one proof from problem set 11

Appendix B. Technique Objectives for a Real Analysis Course

The following table presents the technique objectives for my real analysis course. Each objective is to be used, unless accompanied by the verb "prove" or "est." (establish). "Seq. char." means a sequential characterization: an equivalent version of the definition referred to which is stated only in terms of sequences.

An asterisk represents an objective which can be counted multiple times toward the technique objective count.

topic	level II	level I
basics of $\mathbb{R}$	Cauchy's Criterion Monotone Sequence Property Bolzano-Weierstraß Property Nested Intervals Theorem	an Archimedean principle ϵ-characterization of supremum
limits	seq. char. compute a limsup or liminf	ϵ-δ definition
continuity	topological characterization Intermediate Value Theorem	ϵ-δ definition seq. char.
topology	definition of open set definition of closed set	seq. char. of open set seq. char. of closed set
compactness	topological compactness est. topological compactness	sequential compactness est. sequential compactness Heine-Borel property est. the Heine-Borel property
named theorems		prove an "at infinity" version of a Named Theorem * prove an "infinite limit" version of a Named Theorem *
derivatives	Mean Value Theorem Taylor's Theorem Darboux's Theorem	limit definition Carathéodory's definition
integration	show a 'bad' set is small	Cauchy Criterion
convergence of functions	prove a sequence converges pointwise but not uniformly	est. uniform convergence
series	Root Test	prove the series version of a Named Theorem about sequences *
seq. chars.	formulate & prove a seq. char. *	

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