Online Appendix for
Trust in Risk Sharing: A Double-Edged Sword*

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S.1 The Simple Model: Details and Derivations

In this appendix we provide the detailed derivations underlying Figure 1 in the main text. For concreteness we give a concrete parametric example (which we use to draw the figure), but its qualitative properties hold for any strictly concave utility function.

Let the period utility function is logarithmic, $u(c) = \log(c)$ and the endowment process by given by $h = 1+\epsilon$ and $\ell = 1-\epsilon$ so that expected (average) income in society is $E(y) = 1$, and ϵ measures the degree of income risk (the standard deviation of income). This is also the parameterization analyzed in Krueger and Perri (2006), which allows us to readily compare our results to theirs.

Direct calculations reveal that expected lifetime utility from the autarkic allocation, the first-best insurance allocation (consuming 1 in every period of life), and a stationary allocation with constant transfer $x \in [0, \epsilon]$ are given by

$$\begin{aligned} V^A &= V(x=0) = \log\left[\sqrt{1-\epsilon^2}\right] < 0, \\ V^{FB} &= V(x=\epsilon) = 0, \text{ and} \\ V(x) &= \frac{1}{2}\left[\log(1-\epsilon+x) + \log(1+\epsilon-x)\right] = \log\left[\sqrt{1-(\epsilon-x)^2}\right] \leq 0. \end{aligned} \tag{S.1}$$

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For the parametric example the parameter thresholds for autarky and first-best insurance are available in closed form. Autarky is the only social norm for all discount factors $\beta \leq \underline{\beta} = 2u'(h)/[u'(\ell) + u'(h)] = 1 - \epsilon$, independent of trust π.

At the other extreme, the trust threshold π^{FB} for first-best insurance is given by

$$\pi^{FB} = 1 - \left(\frac{2(1-\beta)}{\beta}\right) \frac{[\log(1+\epsilon)]}{[-\log(1-\epsilon^2)]} < 1. \tag{S.2}$$

If, in addition, $\beta < \underline{\beta}^{FB} := 2\log(1+\epsilon)/[2\log(1+\epsilon) - \log(1-\epsilon^2)] < 1$, then first-best insurance is not a social norm for any trust $\pi \in [0,1]$. Note that $\underline{\beta} < \underline{\beta}^{FB}$, that is, there exist $\beta \in (\underline{\beta}, \underline{\beta}^{FB})$ such that the constrained efficient social norm is non-autarkic but also does not exhibit first-best insurance for any $\pi \in [0,1]$.

We can also analytically characterize the maximally attainable ex-ante lifetime utility (i.e., lifetime utility before being in a coalition) $\overline{F}$, the trust threshold at which this lifetime utility is attained $\overline{\pi}$ (and above which no fixed point to the operator $\mathcal{T}$ exists) and the transfer $\bar{x}$ that implements this value. These are given by

$$\begin{aligned} \bar{x} &= \epsilon + \beta - 1, && \text{(S.3)} \\ \overline{F} &= \frac{1-\beta}{\beta} \log\left[\frac{2-\beta}{1+\epsilon}\right] + \frac{1}{2}\log\left[\beta(2-\beta)\right] && \text{(S.4)} \\ \overline{\pi} &= 1 - \left(\frac{2(1-\beta)}{\beta}\right)\left(\frac{\log(1+\epsilon) - \log(2-\beta)}{\log(\beta(2-\beta)) - \log(1-\epsilon^2)}\right). && \text{(S.5)} \end{aligned}$$

As long as $\beta > \underline{\beta} = 1 - \epsilon$ (otherwise autarky is the only social norm for all π) we have that $\bar{x} > 0$ and $\pi^{FB} < \overline{\pi} < 1$, since then both the numerator and denominator in the last fraction are positive. It is straightforward to see that $\overline{\pi}$ is strictly decreasing in income risk ϵ. The larger is income risk, the smaller is the set of trusts π for which a fixed point exists, and the larger is the set of trusts for which utility needs to be burned to dissuade the high-income agent from leaving the arrangement. Direct calculations also reveal that $\overline{F}$ is strictly decreasing in ϵ and strictly increasing in β.

Figure 1 in the main text plots the value $\Gamma(x)$ of a high-income agent from being in a coalition, and the value $\Psi(x;\pi)$ of a high-income agent from deviating from the coalition, against the stationary transfer x. For the parametric example these are given explicitly by

$$\begin{aligned} \Gamma(x) &= (1-\beta)\log(1+\epsilon-x) + \beta V(x) \text{ and} \\ \Psi(x;\pi) &= (1-\beta)\log(1+\epsilon) + \beta\left[\pi V(x) + (1-\pi)V^A\right] = (1-\beta)\log(1+\epsilon) + \beta F(x), \end{aligned}$$

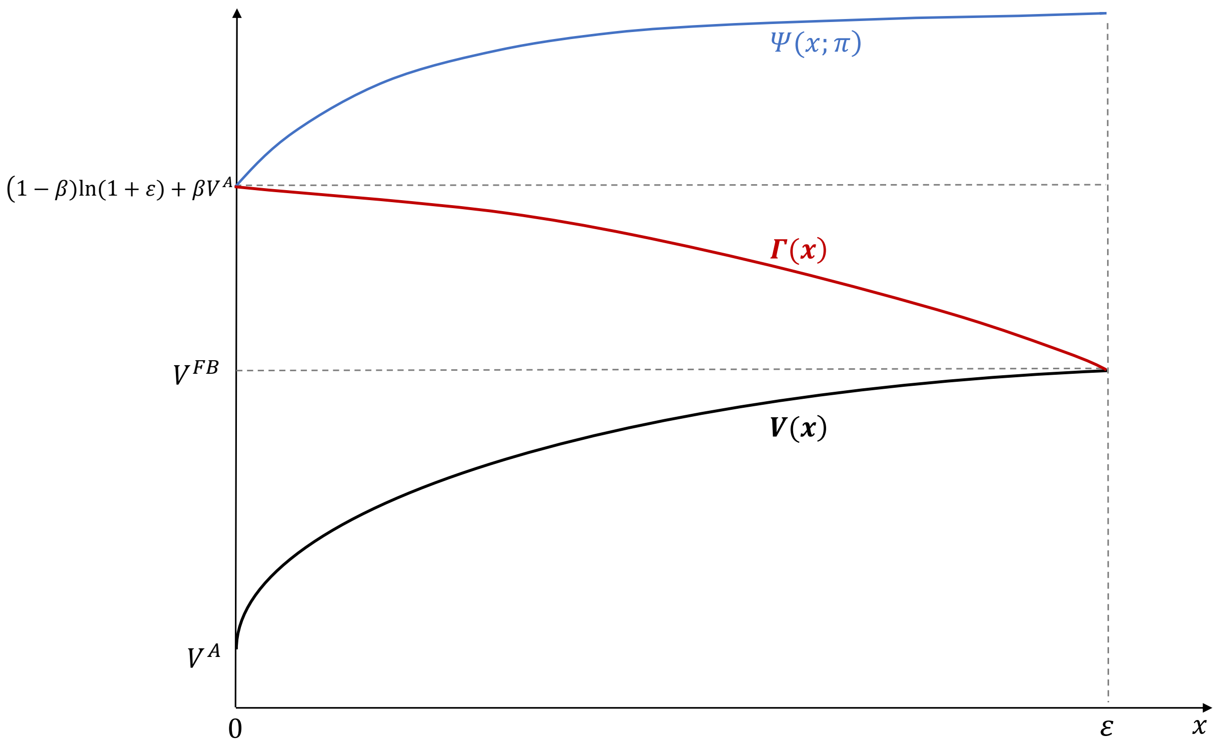

Figure S.1: The functions V, Γ, and Ψ when $\beta < \underline{\beta}$: a graphical representation of autarky.

where $V(x)$ was given in equation $(S.1)$ and

$$F(x) = \pi V(x) + (1-\pi)V^A.$$

Figure 1 in the main text was drawn for parameter values for which first-best insurance is never attainable ($\pi^{FB} < 0$), yet partial insurance is feasible $\beta > \underline{\beta}$. In this appendix, for concreteness we depict the two complementary cases. First, assume $\beta \leq \underline{\beta} = 1 - \epsilon$. Then the value of being a currently high-income agent in a coalition with no risk sharing, $\Gamma(x=0) = (1-\beta)\log(1+\epsilon)+\beta V^A$, exceeds that of first-best insurance, V^{FB}, and is declining in x. Since the value of deviating at $x = 0$ is $\Psi(x = 0; \pi) = \Gamma(x = 0)$ is increasing in x, Figure S.1 shows that the only allocation satisfying the incentive constraint is autarky, and this is the only social norm (and a fixed point). Now assume that $\beta > \underline{\beta} = 1 - \epsilon$, and thus some social norms with positive risk sharing exist. The main text displayed this scenario under the assumption that $\beta \in (\underline{\beta}, \underline{\beta}^{FB})$ and thus $\pi^{FB} < 0$, that is, constrained-efficient social norms always exhibit only partial insurance.

Here we complement the analysis in the main text by displaying, in Figure S.2 the constrained-efficient stationary allocation when $\beta > \underline{\beta}^{FB}$, and thus $\pi^{FB} \geq 0$ in equation

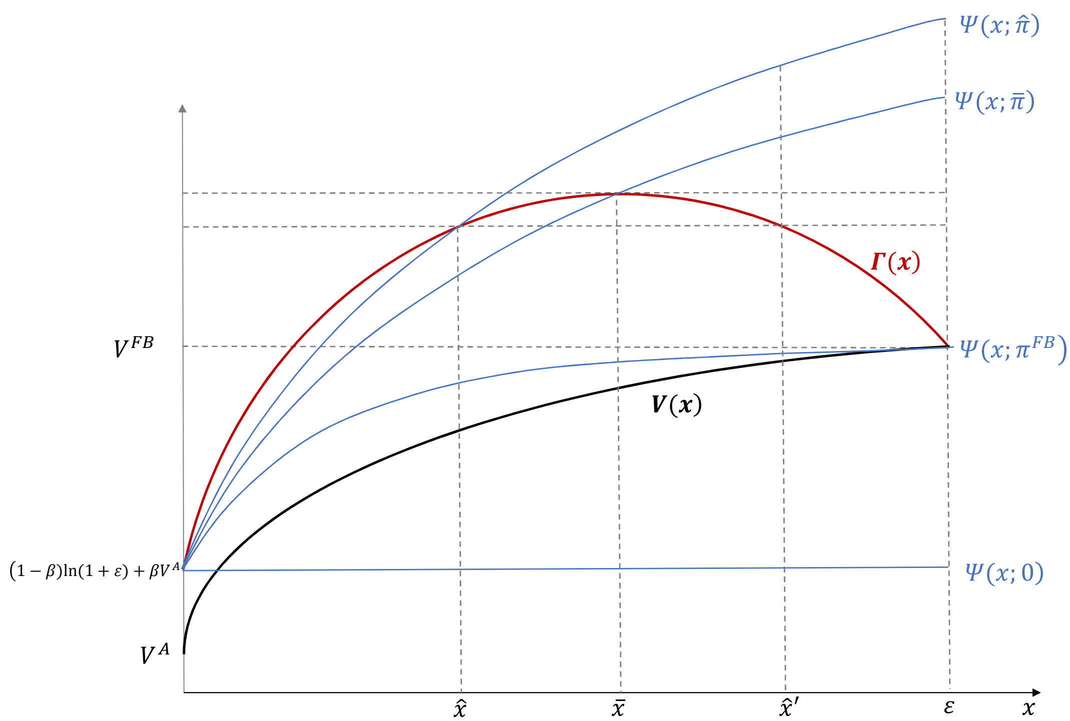

Figure S.2: The functions V, Γ, and Ψ when $\beta^{FB} < \beta$.

$(S.2)$. In this case, for sufficiently low levels of trust π the constrained-efficient social norm displays full consumption insurance.

Figure S.2 traces out how the unique constrained-efficient stationary allocation changes with trust π, as only the outside option $\Psi(x;\pi)$ changes with π. Specifically, this outside option tilts upward as π increases from 0 to 1, around the point $(x=0, \Gamma(x=0))$ for all $\pi \in [0,1]$.

1. For all $\pi \in [0, \pi^{FB}]$ the full-insurance allocation $x(\pi) = \epsilon$ satisfies the incentive constraint since the $\Gamma(x)$-curve lies above the $\Psi(x;\pi)$-curve at $x = \epsilon$. The value of being in a coalition is maximal, at $V(\epsilon)$, and the ex-ante value of being in the unmatched pool, $F(\pi) = \pi V^{FB} + (1-\pi)V^A$ is strictly increasing in π, starting from V^A for $\pi = 0$.[1] Graphically, π^{FB} obtains when the $\Psi(x;\pi)$-curve has just tilted upward enough so that $\Gamma(\epsilon) = \Psi(\epsilon;\pi^{FB})$.

2. As π increases further to a $\pi \in (\pi^{FB}, \overline{\pi})$ and the $\Psi(x;\pi)$-curve tilts further upward, the incentive constraint becomes binding, and the constrained-efficient transfer $x(\pi)$ is

[1] For $\pi = 0$, there is first-best insurance inside a coalition, but since coalitions never form, $F(\pi = 0) = V^A$.

determined by the intersection between the $\Psi(x;\pi)$-curve and the $\Gamma(x)$-curve:

$$\left[1-\frac{\beta(1+\pi)}{2}\right]\log\left(\frac{1+\epsilon}{1+[\epsilon-x(\pi)]}\right)=\frac{\beta}{2}(1-\pi)\log\left(\frac{1-[\epsilon-x(\pi)]}{1-\epsilon}\right) \tag{S.6}$$

The risk-sharing transfer $x(\pi)$ is strictly decreasing in π and the ex ante value $F(\pi)\equiv F(x(\pi))$ can be read off from the y-axis and is strictly increasing in π, even though expected utility of being matched in a coalition, $V(x(\pi))$ is strictly decreasing in π.

3. As π reaches $\overline{\pi}$ given in equation ($S.5$) the intersection of the $\Psi(x;\overline{\pi})$-curve and the $\Gamma(x)$-curve occurs at $\bar{x}=\epsilon+\beta-1$ and the maximally attainable ex ante utility of being unmatched is given by $\overline{F}$ given in equation (S.4).

4. Finally, as π increases further beyond $\overline{\pi}$, say to $\hat{\pi}\in(\overline{\pi},1]$, the $\Psi(x;\hat{\pi})$-curve and the $\Gamma(x)$-curve intersect, but at an $\hat{x}<\bar{x}$, with associated ex-ante value $\hat{F}$, and thus the allocation $\hat{x}$ satisfies the incentive constraint if the outside option is given by $\hat{F}$. However, a coalition faced with this outside option $\hat{F}$ will choose allocation $\hat{x}'$ with implied value $\hat{F}'>\hat{F}$, and thus $(\hat{x},\hat{F})$ is not a fixed point for $\hat{\pi}$. For such high trust $\hat{\pi}>\overline{\pi}$, the full model analysis will show that the ex-ante value will remain at $\overline{F}<\pi V(\bar{x})+(1-\pi)V^A$, and will be implemented by a non-stationary allocation (which cannot be depicted in the figure) that "burns" utility relative to the better social norm $\bar{x}$ which, however, if implemented, would result in an outside option so high that the coalition itself is left with an empty set of resource- and incentive-compatible allocations.

S.2 Model Extensions

In this appendix we discuss two extensions of our model. In the first we consider a more general model of temporary delays to agreement after an initial failure to successfully form a coalition. In the second we extend out model to allow for production.

S.2.1 Temporary Delay

We have assumed that a deviating coalition succeeds with probability π and is in permanent autarky with complementary probability. We now assume that a failure to form a coalition is followed by $T\geq 1$ periods of autarky before another attempt can be made (so that if $T=1$, a new attempt can be made in the next period after a failure). Under this assumption, after a deviation, coalition formation always eventually occurs. For fixed trust π a reduction of T

increases the outside option. We now argue that the extension with a delay of T is equivalent to our original model with trust

$$\pi^{\dagger} := \frac{\pi}{1-(1-\pi)\beta^T}.$$

Suppose $\mathbb{c}^{\dagger}$ is an efficient allocation in the model with T-period delay. Then, the value of the outside option after deviating satisfies

$$W^d = \pi W^0(\mathbb{c}^{\dagger}) + (1-\pi)[(1-\beta^T)V^A + \beta^T W^d],$$

that is,

$$W^d = \pi^{\dagger} W^0(\mathbb{c}^{\dagger}) + (1-\pi^{\dagger})V^A.$$

It is easy to verify that since $\mathbb{c}^{\dagger}$ is an efficient allocation in the model with T-period delay, it must also be an efficient allocation in our original model for trust $\pi^{\dagger}$.

With finite exclusion, all agents are eventually in a risk-sharing arrangement, irrespective of the level of trust. However, the level of risk-sharing is declining in trust.

S.2.2 Risk Sharing and Production

We now briefly discuss how to extend our model to a production economy where output is produced and consumption is allocated within coalitions we will call production clubs, or firms for short. Output y_t produced by agent at time t depends upon idiosyncratic productivity $e_t \in E = \{e_\ell, e_h\}$ and labor effort l_t.

$$y_t = e_t l_t$$

Individual preferences are given by

$$(1-\beta)\mathbb{E}\Big\{\sum_{t=1}^{\infty}\beta^t U(c_t, l_t)\Big\},$$

and labor effort is bounded by the unit interval, so $l_t \in [0,1]$. All other aspects of the environment are the same as in the endowment economy studied thus far.

As before, risk-sharing incentives lead to continuum-sized firms being efficient, just as in our endowment economy. Since this implies that there is no aggregate output risk within a firm, an allocation within a continuum-sized firm are sequences of consumption and labor effort, both functions of the individual productivity history, $\{c_t(e^t), l_t(e^t)\}$.

In the special case in which labor is inelastically supplied at 1, and preferences are separable in consumption and labor, the efficient allocation of our model becomes essentially the same as in the endowment case, with endowment income $y \in \{\ell, h\}$ replaced by production income $y \in \{e_\ell \times 1, e_h \times 1\}$. This is the content of the next proposition.

Proposition S.1 *Suppose flow utility $U(c_t, l_t) = u(c_t) - v(l_t)$ is separable between consumption and labor, $(e_\ell, e_h) = (\ell, h)$, and $u'(y_h)y_\ell \geq v'(1)$.*

1. *There exists an efficient allocation with a consumption allocation that is identical to that in the endowment economy with $c(e^t) = c(y^t)$ and labor equal to $l_t(e^t) = 1$.*

2. *The payoff to forming a firm is the same as in the coalition payoff in the endowment economy, net of the cost of labor effort:*

$$(1-\beta)\mathbb{E}\Big\{\sum_{t=1}^{\infty} \beta^t [u(c_t(e^t)) - v(l_t(e^t))\Big\} = W^0(c) - v(1).$$

3. *The largest probability of successfully forming a firm for which there is a fixed point is still $\overline{\pi}$ from the endowment economy, however the associated highest feasible outside option is $\overline{F} - v(1)$.*

This proposition follows from the fact that the rankings of consumption sequences is unaffected by subtracting a constant labor cost in each period. For $\pi > \overline{\pi}$ utility-burning needs to occur in an efficient allocation, and while this can be done just as in the endowment case, richer possibilities involving the labor allocation emerge in the production economy.

The key to the previous proposition is that the within-firm consumption-labor allocation can be solved sequentially. In a first step the optimal labor allocation is determined, and in a second step the consumption risk-sharing allocation is chosen, taking as given the stochastic income process from the first stage. For a general utility function where labor is interior both consumption and labor are determined jointly.

An exception are utility functions without income effects on labor supply. For example, suppose households have Greenwood, Hercowitz, and Huffman (1988) preferences of the form

$$U(c,l) = \frac{1}{1-\gamma}\left\{c - \Psi \frac{l^{1+\theta}}{1+\theta}\right\}^{1-\gamma}$$

then the optimal labor allocation is determined by $l_t(e^t) = (e_t/\Psi)^{1/\theta}$ if Ψ is sufficiently large relative to e_t so that $l_t(e^t) < 1$. Now idiosyncratic income is given as $y(e^t) = \frac{e_t^{1+1/\theta}}{\Psi^{1/\theta}}$ and is efficiently shared within the firm as before, leading to a consumption allocation similar to the endowment economy. However, now we need to adjust the payoffs to take account of the differential labor utility costs. For example, the decay condition (23) in Proposition 3 becomes

$$\frac{\left(c(e^t) - (e_t/\Psi)^{(1+\theta)/\theta}\right)^{-\gamma}}{\left(c(e^{t+1}) - (e_{t+1}/\Psi)^{(1+\theta)/\theta)}\right)^{-\gamma}} = \delta_{t+1}.$$

Finally, it is easy to accommodate the notion that firms can realize increasing returns to scale, up to a point, in the size of its workforce, and that the production coalitions we model partially form not only for risk sharing purposes, but also for production efficiency purposes. Suppose that individual output within a firm is now given by

$$y_t = ze_tl_t$$

where $z = z(x)$ is a positive and weakly increasing function of the size x of the workers of the firm, with $z(x) = 1$ for $x \geq X$. That is, for firms larger than size $X < \infty$, which include those with an infinite number or a continuum of members, $z(x) = 1$. When $z(0) < 1$, then producing in autarky involves not only a loss in consumption smoothing but also a reduction in productivity. This again leads to a consumption allocation that has the same characteristics as in the endowment economy, but with a reduction in the value of autarky. With period utility that is separable and CRRA in consumption the utility from autarky is scaled to $u(z(0))V^A(y)$.[2] Scaling down the utility from autarky raises π^{FB} and $\overline{\pi}$, the trust at which first-best insurance can be sustained and the threshold trust for which the fixed-point exists and utility burning is unnecessary. Thus, while the qualitative features of the analysis are unaffected by productivity benefits of large coalitions, quantitatively such production coalitions can provide better insurance when formed.

Our model of production clubs can qualitatively account for a number of well known features of the data. In the context of the literature on trust, Fukuyama (1995, p. 309, 312) asserts that while “there continues to be a steady proliferation of interest groups of all sorts in American life ... communities of shared values whose members are willing to subordinate their private interests for the sake of larger goals of the community ... have become rarer.” This is consistent with the prediction of our model that more coalitions forming goes hand in hand with shallower cooperation within coalitions. On the issue of risk sharing within a firm,

[2] If the disutility of labor such that it is always efficient to supply a unit of labor in autarky for all levels of idiosyncratic productivity, then this simply shifts down the autarky payoff in the production economy relative to the endowment economy and is given by

$$(1-\beta)[u(z(0)y) - v(1)] + \beta\mathbb{E}_{y'}[u(z(0)y') - v(1)] = u(z(0))V^A(y) - v(1).$$

Guiso, Pistaferri, and Schivardi (2005) find that while temporary shocks are well-insured, permanent ones are not. This is consistent with our model, since a permanent shock to a worker's income would rescale their outside option and hence lead to a permanently different consumption ladder.[3]

S.3 Numerical Examples and Comparative Statics

In this section we provide numerical examples illustrating, in Subsection S.3.1, how the trust threshold for utility burning $\bar{\pi}$ changes with patience β away from $\underline{\beta}$, and, in Subsection S.3.2, how the dynamics of strong social norms evolves towards the limit stationary ladder. Subsection S.3.3 provides the details of the computational algorithm employed to derive these numerical results.

S.3.1 Insurance Possibilities and Trust Thresholds π^{FB} and $\bar{\pi}$

We first numerically characterize the threshold trust values for full insurance, π^{FB} and for the existence of strong social norms (and thus for the absence of utility burning), $\bar{\pi}$. These calculations complement the third part of Proposition 4 in the main text where we proved that near $\underline{\beta}$, the threshold $\bar{\pi}$ is less than one and thus utility burning must occur for sufficiently high values of trust.

Figure S.3 plots the threshold values for π^{FB} and $\bar{\pi}$ against the discount factor β, for an income process with $h = 1.25$ and $\ell = 0.75$, and logarithmic period utility, $u(c) = \log(c)$. This parameterization implies that $\underline{\beta} = u'(h)/u'(\ell) = 0.75/1.25 = 0.6$.

The figure demonstrates that for discount factors $\beta \leq \underline{\beta}$ the constrained-efficient allocation is autarkic independent of trust π. For values of $\beta > \underline{\beta}$, in contrast, the constrained-efficient allocation changes qualitatively as trust π increases. Take $\beta = 0.9$ for concreteness: for low values $\pi \leq \pi^{FB}$ first-best insurance can be sustained, for intermediate values $\pi \in (\pi^{FB}, \overline{\pi}]$ there is partial risk sharing but no utility burning, and for $\pi > \overline{\pi}$ the constrained-efficient social norm requires utility burning. Importantly, this numerical example shows that for all $\beta < 1$, the threshold trust level $\overline{\pi}(\beta)$ above which utility burning needs to occur as part of the constrained-efficient allocation is always less than one, a feature that we have robustly found through many parameterizations we have explored.

[3] With homothetic preferences, a permanent multiplicative shock to productivity for a (positive measure) subset of agents would simply scale these agents' consumption allocation by the permanent shock, since these agents with the positive shock can always secede and guarantee themselves the scaled consumption process.

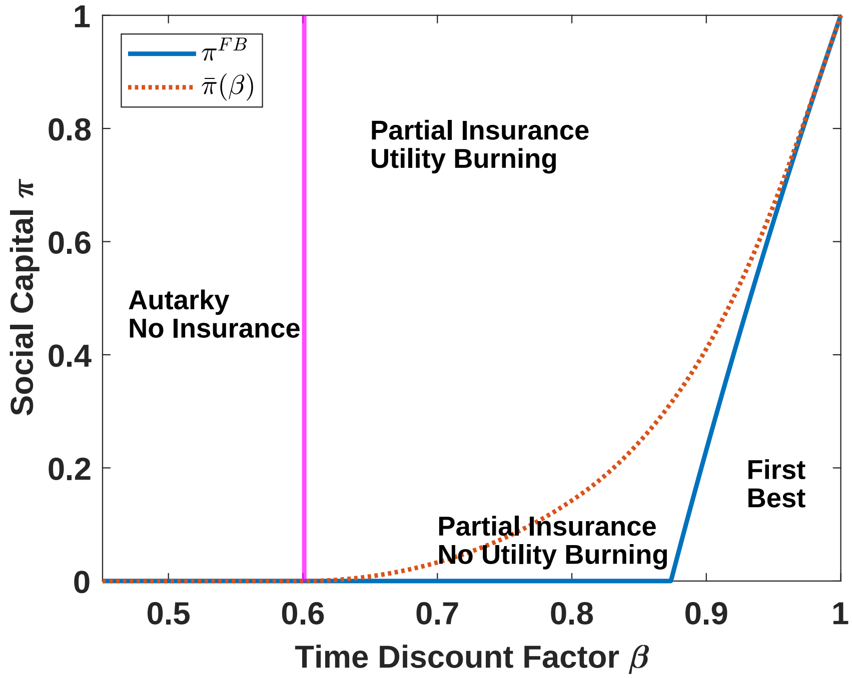

Figure S.3: Insurance possibilities as a function of (β, π), for $h = 1.25$, $\ell = 0.75$, $u = \log$.

S.3.2 The Dynamics of Strong Social Norms

In this subsection we present results for an illustrative set of examples to convey the qualitative properties of strong social norms. Throughout this section we assume a CRRA period utility function. This functional form implies that equation (23) characterizing strong social norms can be written as

$$\forall y^t, \mathbb{c}(y^t\ell) > c_\ell(F) \implies \frac{\mathbb{c}(y^t)^{-\gamma}}{\mathbb{c}(y^t\ell)^{-\gamma}} = \delta_{t+1},$$

for some $\delta_{t+1} < 1$. Since $\delta_{t+1} < 1$, and defining $g_{t+1} := (\delta_{t+1})^{1/\gamma} < 1$,

$$\forall y^t, \mathbb{c}(y^t\ell) > c_\ell(F) \implies \mathbb{c}(y^t\ell) = g_{t+1}\mathbb{c}(y^t).$$

By Proposition 3 strong social norms have the form of a sequence of consumption ladders (as defined in Definition 8), where the period t-ladder is determined by an initial consumption after the high income $y = h$ realization, $\mathbb{c}_t(h)$, and then a decreasing sequence of lower consumptions $g_{t+1}\mathbb{c}_t(h), g_{t+1}g_{t+2}\mathbb{c}_t(h), \ldots$, until the lower bound $c_\ell(F)$ is reached (after $L-1$

realizations of ℓ). The strong social norm converges to a stationary ladder and associated constant decay rate $g_t = g_{t+1} = g$.

With these observations from our theoretical results in hand, the computation of strong social norms with associated outside option $F \in (V^A, \overline{F}]$ (and thus for trust π associated with that outside option) proceeds as follows.[4] The algorithm first computes a stationary consumption ladder and associated consumption decay rate g that satisfies the h-incentive-feasibility constraint associated with F with equality (as well as the resource constraint and the ℓ-incentive-feasibility constraint with equality for those at the very bottom of the ladder). It then determines the full dynamic strong social norm by imposing convergence to the stationary ladder in finite (but potentially long) time.

Figure S.4 plots the dynamics of the efficient consumption allocation with $u(c) = \log(c)$, incomes are $(\ell, h) = (0.75, 1.25)$, and the discount factor is chosen as $\beta = 0.9$. The level of trust is set to $\pi = \overline{\pi} = 0.41$ so that the value of the outside option is given by $F = \overline{F}$. Table 1 provides additional summary statistics for the allocation in this parameterization, as well as for alternative values of (β, γ) to display the comparative statics of the model with respect to its preference parameters (the values of $\overline{F}$ and $\overline{\pi}$ changes with (β, γ)).

From Figure S.4 we observe that as the transition unfolds, consumption spreads out over time, and eventually converges to the stationary ladder, which for this parameterization has five consumption steps. Consumption insurance worsens over time but remains positive: for high income agents the outside option is binding, but they consume substantially less than their income h (indicated by the upper dashed line) and thus provide insurance to low-income agents. Initially low income agents consume significantly more than their income (lower dashed line), and also more than implied by a binding outside option, $c_\ell(\overline{F})$. Over time those with continuously low income see their consumption drift down until the outside option binds and $c = c_\ell(\overline{F})$. This occurs in period four of the transition.

The strong social norm can generate high initial consumption insurance because the allocation does not inherit any implicit promises to past high income types. As time evolves, the consumption level of $c(\ell^t)$ declines as the burden of efficient smoothing of consumption to past high income types makes consumption scarcer. The allocation also becomes statically inefficient since agents with the same current income receive different consumption levels. Finally, the figure shows that although we do not force convergence to the stationary ladder until period 10 (the last period of the transition phase prior to imposing a stationary ladder) in this example, effectively allocations have converged to the stationary ladder by period four of the transition. Expanding the length of the transition yields utility gains that are indistinguishable from zero. Thus, although theoretically convergence to the stationary

[4]The details of the computational procedure are described in Subsection S.3.3

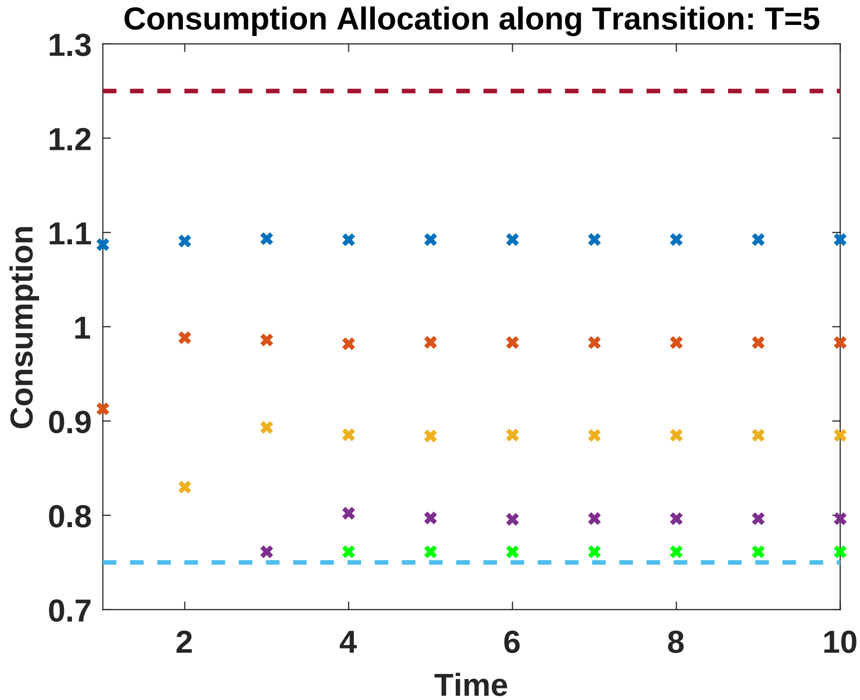

Figure S.4: Consumption allocation along transition, with $\ell = 0.75$ (indicated by lower dashed horizontal line), $h = 1.25$ (upper dashed horizontal line), $\beta = 0.9$, $\pi = 0.41$, and $\gamma = 1$.

ladder is only asymptotic, our example suggest that numerically convergence occurs very rapidly; an observation shared by all examples we have computed.

Table 1 contains summary statistics of strong social norms along the transition for alternative parameterizations of the model. Focus first on the benchmark case in the first column: we observe that the consumption allocation a coalition can implement improves significantly (worth 0.94% of consumption) on the outside option, by providing insurance to initially poor agents, but also needs to leave significant insurance opportunities unexploited (worth 0.63% of consumption relative to first-best insurance). Insurance gets worse over time as expected period utility falls and consumption dispersion rises over time.[5] As households become more patient (higher β) and more risk-averse (higher γ), the strong social norm gets closer to first-best insurance, but the gains from coalition risk sharing relative to the outside option become smaller. The stationary ladder has more steps and the support of the consumption distribution tightens. We also observe that increased patience (higher β), elevates the gains of coalition risk sharing (compared to the outside option) mostly through an improvement of the stationary ladder. An increase in risk aversion (larger γ), in contrast, leads to better

[5]We only display the first two periods, relative to the stationary ladder.

Statistic	$\gamma=1$ $\beta=0.9$	$\gamma=1$ $\beta=0.95$	$\gamma=2$ $\beta=0.9$	$\gamma=2$ $\beta=0.95$
$V^{FB}/V(\overline{F})$ in %	0.63%	0.22%	0.45%	0.12%
$V(\overline{F})/\overline{F}$ in %	0.94%	0.71%	1.24%	0.80%
$\overline{\pi}$	0.41	0.66	0.69	0.85
$c_\ell(\overline{F})$	0.761	0.767	0.776	0.782
c_h	1.092	1.049	1.050	1.025
Steps	5	8	7	12
$\frac{EU(c_1)}{EU(c_\infty)}$ in %	0.28%	0.11%	0.25%	0.07%
$\frac{EU(c_2)}{EU(c_\infty)}$ in %	0.05%	0.05%	0.11%	0.03%
$Var(c_\infty)$	0.01	0.004	0.004	0.001
$\frac{Var(c_1)}{Var(c_\infty)}$	0.62	0.55	0.55	0.52
$\frac{Var(c_2)}{Var(c_\infty)}$	0.94	0.81	0.81	0.77

Table 1: Summary Statistics of the Transition

Notes: Ratios of (lifetime) utilities are converted into consumption equivalent variation and give the percentage increase in consumption (uniform across all states or histories) required to equalize period (or lifetime) utility across the two alternatives. The first two lines measure the welfare loss from imperfect consumption insurance relative to first-best insurance, and the welfare gain of coalition allocations relative to the outside option. The second panel provides summary statistics of the stationary ladder, and the third and forth panels show how expected utility and consumption insurance declines over time.

risk sharing both because of an improved stationary ladder and longer initial insurance and thus slower convergence to the ladder.

S.3.3 Computational Details for Section S.3

In this subsection we provide the details of how we compute strong social norms. Section S.3.3.1 describes how to compute a stationary ladder that delivers an outside option $F \in (V^A, \overline{F})$. Section S.3.3.2 describes how to determine the value of $\overline{F}$ (and the associated threshold trust value $\bar{\pi}$) together with the stationary ladder attaining it. Finally, Section S.3.3.3 describes the calculation of an entire dynamic efficient consumption allocation converging to a stationary ladder.

S.3.3.1 Stationary Ladder

For a *fixed* F, a stationary ladder $c_* = (c_*(h), gc_*(h), g^2c_*(h), \ldots, c_\ell)$ that satisfies feasibility and F-incentive compatibility for high income individuals (henceforth h-incentive compatibility) with equality as well as F-incentive compatibility for individuals at the bottom of the stationary ladder (henceforth ℓ-incentive compatibility) with equality is fully characterized by the upper and lower bound of consumption $(c_*(h), c_\ell)$, the decay rate g and the length of the ladder L. These values, all functions of a given $F \in (V^A, \overline{F})$, are calculated as follows:

1. Determine the unique consumption floor $c_\ell = c_\ell(F)$ from Lemma 8, i.e.,

$$u(c_\ell(F)) = u(\ell) + \beta \left(F - V^A\right)$$

and recall the value of the outside option for the high income agents is

$$W^F(h) := (1-\beta)u(h) + \beta F.$$

2. A stationary ladder attaining F is then determined by three equations in the three unknowns $c_*(h), g, L$ from

$$L = \max\left\{k : g^{k-1} c_*(h) > c_\ell(F)\right\}, \tag{S.7}$$

$$\frac{1}{2}\sum_{t=0}^{L-1}\left(\frac{1}{2}\right)^t c_*(h) g^t + \left(\frac{1}{2}\right)^L c_\ell(F) = \bar{y}, \tag{S.8}$$

and, using $W(h, c_*) = W^F(h)$ in equation (26) characterizing lifetime utility from a stationary consumption ladder,

$$W^F(h) = \left(1 - \frac{\beta}{2}\right)\left[\sum_{k=0}^{L-1}\left(\frac{\beta}{2}\right)^k u(c_*(h) g^k)\right] + \left(\frac{\beta}{2}\right)^L u(c_\ell(F)). \tag{S.9}$$

This system of equations can be reduced to one non-linear equation in the unknown decay rate $g \in [\ell/h, 1]$. Use equation (S.7) to solve for the unique length of the ladder $L(g, c_*(h))$ given g, and then equation (S.8) to solve for the unique entry level consumption $c_*(h)$ (exploiting the fact that the period utility function is of CRRA variety), and insert both entities into equation (S.9) to obtain one equation in the unknown consumption decay rate g. The result of solving this one-dimensional nonlinear equation in g is a stationary ladder summarized by $(c_*(h)(F), g(F), L(F))$ as a function of the outside option F.

In general the stationary ladder associated with an outside option F need not be unique, although it is for $F = \overline{F}$, as we have seen in Section 7 of the main text. Computationally, since g must be bounded between $g = 1$ (no consumption decay, as in the full-insurance allocation) and $g = \ell/h$ (the consumption decay in the autarkic allocation), it is straightforward to determine all solutions to this one-dimensional nonlinear equation.

However, to better understand conceptually the potential multiplicity of stationary ladders and to determine which of the potentially several ladders is the relevant limit stationary ladder of the strong social norm (for the given F), it is instructive to proceed as follows. Instead of calculating the consumption decay rate g (and the associated $(c_*(h), L)$) as a function of F, in step 2 above we reverse the order and calculate, for a given stationary consumption ladder decay rate $g \in (\ell/h, 1)$, the outside option $F(g)$ attained by this g (and the associated consumption ladder) and plot F against $g \in [\ell/g, 1]$.

Numerically, we find that the mapping $F(\,\cdot\,)$ is hump-shaped with a maximum at $\bar{g} := \beta^{1/\gamma} < 1$ that delivers the maximum value $\overline{F}$.[6] That the decay rate at $F = \overline{F}$ is given by $\bar{g} = \beta^{1/\gamma}$ can easily be shown theoretically and follows directly from the first-order condition of the program defining in $\overline{F}$ in (29) of the main text. The reason for the hump-shape of $F(\cdot)$ is as follows. Start at $g = 1$, and thus a constant consumption allocation with first-best insurance, and now lower g infinitesimally. Individuals with current income $y = h$ strictly prefer a more front loaded consumption allocation even though it entails more consumption risk in the future. As g initially falls from $g = 1$, both $W(h, c_*)$ and $c_*(h)$ increase, which in turn leads the outside option $F(g)$ to increase as g falls. At $g = \beta^{1/\gamma}$ the optimal front loading is attained from the perspective of the current h types; by reducing g further the associated increased future consumption risk more than offsets the higher current consumption $c_*(h)$ chosen to satisfy the resource constraint. Thus $W(h, c_*)$ and $F(g)$ decline as g falls beyond $g = \beta^{1/\gamma}$.

We cannot prove that $F(g)$ is hump-shaped in g but always found this to be the case in our numerical examples. This implies, in particular, that for any $F < \overline{F}$ there are two associated stationary ladders that deliver the same outside option F, one with little risk sharing ($g < \bar{g}$) and one with more risk sharing ($g > \bar{g}$). Since the algorithm for computing a dynamic strong social norm is based on the convergence of the allocation to a stationary ladder, it is important to know which ladder to pick, for a given $F < \overline{F}$.

In Lemma 15 we have shown that although there might be multiple candidate stationary ladders, the one the strong social norm converges to asymptotically is the one with the smallest entry level of consumption $c_*(h)$ and thus the slowest consumption decay and the largest extent of risk sharing. Thus, for the purpose of the computation of dynamic strong social norms we restrict attention to stationary ladders with decay rates $g \in [\bar{g}, 1]$.

[6] This result accords well with the results in the simple model where the outside option F from a stationary allocation was also hump-shaped in the extent of consumption insurance (which in the simple model was measured by the size of the transfer x), with the maximum $\overline{F}$ attained at $\bar{x}$.

S.3.3.2 Determination of the Outside Option $\overline{F}$

To determine $\overline{F}$ we proceed as follows: At $F = \overline{F}$, Proposition 6 implies that there is a unique stationary ladder satisfying h-incentive compatibility and this ladder solves (29), so we know that the consumption decay rate is given by

$$g(\overline{F}) = \beta^{1/\gamma}.$$

In effect, $\overline{F}$ is the peak of the $F(\,\cdot\,)$ map discussed above, and is reached at $g = \bar{g}$. Since the value of $\overline{F}$ itself is unknown, we have to determine the lower consumption floor $c_\ell = c_\ell(\overline{F})$ jointly with $\overline{F}$, $c_*(h)$, and L. The relevant equations, with $g = g(\overline{F}) = \beta^{1/\gamma}$ are

$$u(c_\ell) = u(\ell) + \beta\left(\overline{F} - V^A\right), \tag{S.10}$$

$$\bar{y} = \frac{1}{2}\sum_{t=0}^{L-1}\left(\frac{1}{2}\right)^t c_*(h)g^t + \left(\frac{1}{2}\right)^L c_\ell, \tag{S.11}$$

$$L = \max\{k : g^{k-1}c_*(h) > c_\ell\}, \quad \text{and} \tag{S.12}$$

$$(1-\beta)u(h) + \beta\overline{F} = \left(1 - \frac{\beta}{2}\right)\left[\sum_{k=0}^{L-1}\left(\frac{\beta}{2}\right)^k u(c_*(h)g^k)\right] + \left(\frac{\beta}{2}\right)^L u(c_\ell). \tag{S.13}$$

The algorithm to determine $\overline{F}$ is then a slightly modified version of the procedure from the previous subsection, with $\overline{F}$ replacing g as the unknown to be computed, and are identical to the computations we carry out when solving for $F(g)$ for a given $g \neq \bar{g}$.

1. Guess $\overline{F} \in (V^A, V^{FB})$.

2. For a given $\overline{F}$:

   (a) Solve for c_ℓ from (S.10).

   (b) Jointly solve for $(c_*(h), L)$ from (S.11) and (S.12).

   (c) Calculate the right side of (S.13).

3. Solve $\overline{F}$ such that (S.13) holds.

Finally, once $\overline{F}$ is computed, we can determine $\bar{\pi}$ from equation (24).

S.3.3.3 Computation of the Transition

As discussed at the beginning of this section, the computational procedure solves for the strong social norm for a given F, imposing the stationary ladder from an exogenously specified period T. We now describe the computation of the allocations for fixed T and fixed outside option $F \in [V^A, \overline{F}]$. We take as given the stationary ladder associated with F, summarized by $(c_*(h)(F), g(F), L(F))$, including the lifetime continuation utilities $V_{i,*}(F)$ from being in step i of the stationary ladder, as described in the previous two subsections.[7] As described at the beginning of this section, the algorithm calculates consumption in three phases.

In the first $t \leq T$ periods the algorithm picks time-varying consumption of agents with currently high income (and so have binding incentive constraints), $(\mathbb{c}_t(h))_{t=1}^T$ and uses the resource constraints and the fact that agents without binding constraints have common consumption decay rates (or consume the lower bound consume $c_\ell(F)$) to pin down the remainder of the consumption allocation. In a second phase, from $t = T+1, ..., T+L(F)$ the allocation blends into the stationary ladder: all agents with high income consume according to the stationary ladder, and all households with low income drift down from consumption in the previous period at a common (across individuals, but time-varying) decay rate g_t. Finally, for all $t > T + L(F)$, the allocation coincides with the stationary ladder. More precisely, the algorithm works as follows:

1. Guess $(\mathbb{c}_t(h))_{t=1}^T \in (\bar{y}, h)^T$.

2. Calculate the consumption allocation implied by this guess, imposing the characterization of a strong social norm from Proposition 3: the h-incentive-feasibility constraint holds with equality in every period, and all agents with low income either have non-binding constraints and their consumption decays at a common rate or they consume c_ℓ. The implied consumption allocations $(c_{i,t})_{i=0}^t$ for all $t = 1, \ldots, T, T+1, \ldots, T+L(F)$, are calculated as follows, where i again indicates the position on the consumption ladder:

   (a) Set

$$\begin{aligned} & c_{0,t} = \mathbb{c}_t(h) \text{ for } t = 1, \ldots, T, \\ \text{and} \quad & c_{0,t} = c_*(h)(F) \text{ for } t = T+1, \ldots, T+L(F). \end{aligned}$$

   (b) For $t = 1$, determine $c_{1,1}$ from

$$\frac{1}{2}\left[c_{0,1} + c_{1,1}\right] = \bar{y}.$$

[7] The only part that distinguishes the calculations for $F < \overline{F}$ and $F = \overline{F}$ is the calculation of the stationary ladder(s), and in case of $F < \overline{F}$, the selection of the "right" ladder.

(c) For $t = 2, \ldots, T$, determine the consumption decay rates $(g_t)_{t=2}^T$ recursively (beginning with $t = 2$) as follows:

The consumption decay g_t solves

$$\frac{1}{2}\sum_{i=0}^{t-1}\left(\frac{1}{2}\right)^i c_{i,t} + \left(\frac{1}{2}\right)^t c_{t,t} = \bar{y},$$

where for all $i = 1, \ldots, t$,

$$c_{i,t} = \max\{g_t c_{i-1,t-1}, c_\ell(F)\}.$$

For each t, g_t is determined by one equation. The equations are solved forward in time since the allocations $\{c_{i,t}\}$ require knowledge of allocations $\{c_{i-1,t-1}\}$.

(d) For $t = T+1, \ldots, T+L(F)$, part of the consumption allocations are on the stationary ladder. For each $t = T+1, \ldots, T+L(F)$, the consumption decay g_t solves

$$\frac{1}{2}\sum_{i=0}^{t-1}\left(\frac{1}{2}\right)^i c_{i,t} + \left(\frac{1}{2}\right)^t c_{t,t} = \bar{y},$$

where

$$c_{i,t} = \begin{cases} g^i c_h(F), & \text{for } i = 1, \ldots, t-T-1, \\ \max\{g_t c_{i-1,t-1}, c_\ell(F)\}, & \text{for } i = t-T, \ldots, t. \end{cases}$$

3. For a given guess $(\mathbb{c}_t(h))_{t=1}^T$, the previous step delivers the entire allocation $(c_{i,t})_{i=0}^t$ for periods $t = 1, \ldots, T, T+1, \ldots, T+L(F)$. From date $t = T+L(F)+1$ on the consumption allocation coincides, by assumption, with the stationary ladder. Now we need to determine $(\mathbb{c}_t(h))_{t=1}^T$. These values must yield a consumption allocation that delivers the outside option $W^F(h)$ for all $t = 1, \ldots, T$. Construct the lifetime utility in period t after the history $y^{t-1-i}h\ell^i$, $V_{i,t}$, from the consumption allocation computed in the previous step. This can be done recursively, going backward in time. Lifetime utilities are given by, for each $t = T+L, \ldots, 1$ (working backwards in time) and all $i = 0, \ldots, t$,

$$V_{i,t} = (1-\beta)u(c_{i,t}) + \frac{\beta}{2}\left[V_{0,t+1} + V_{i+1,t+1}.\right]$$

Note that these calculations are the same before and in the blended phase, because $V_{0,t}$ is a function of $V_{i,t+i}$ for $i = 1, \ldots, L$, with $V_{L,T+L} = (1-\beta)u(\ell) + \beta F$ and $t \leq T+L$. The only role the consumption levels from the stationary ladder play is in step 2 above in determining $c_{i,t}$ via feasibility.

Finally we need to check whether the entry consumption levels $(\mathbb{c}_t(h))_{t=1}^T$ are such that the resulting consumption allocation hits the outside option for each $t = 1, \ldots, T$

$$V_{0,t} = (1-\beta)u(h) + \beta F.$$

If yes, we are done. If not, go back to step 1 and adjust the guess for $(\mathbb{c}_t(h))_{t=1}^T$.

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