Shaping Inequality and Intergenerational Persistence of Poverty: Free College or Better Schools*

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Abstract

In this paper we evaluate alternative government education policies to encourage college attendance and completion, such as making college free and improving funding for primary and secondary public schooling. To do so, we construct a general equilibrium overlapping generations model with intergenerational linkages and a multi-stage human capital production process during childhood and adolescence with both parental time and resource investments as well as government schooling inputs investments. The model features rich cross-sectional heterogeneity, and specifically, distinguishes between single and married parents, and is disciplined by US household survey data on income, wealth, education and time use. Studying the transitions induced by unexpected policy reforms we show that a) both sets of reform generate significant welfare gains, but transitions take time so that these welfare gains take time to materialize, b) general equilibrium effects are important: the decline in college wages induced by larger college attainment dampens the increase in the college share and reduces the welfare gains, relative to a partial equilibrium version of the model where factor prices remain fixed, c) for a fixed budget of the reforms, spending it on better schools generates more widespread welfare gains that the free college reform, especially at the lower end of the socio-economic spectrum, and d) the optimal policy combination splits the budget between both policies: better schools bring the average level of human capital at age 18 up and tuition subsidies make college affordable even for children from poorer parental backgrounds.

Keywords: education spending, public transfers, welfare benefits, inequality, poverty, intergenerational persistence

J.E.L. Codes: D15, D31, E24, I24

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1 Introduction

In international comparison, the U.S. displays low intergenerational socio-economic mobility, with especially high persistence at the bottom of the income distribution. Achievement gaps between children of different socio-economic backgrounds appear early in life and persist into adulthood. Skill and achievement gaps during adolescence (i.e. before labor market entry) have been identified as a key factor determining differences in later life economic outcomes (see, e.g., Keane and Wolpin (1997); Carneiro and Heckman (2002); Huggett et al. (2011)), and a variety of policies have been proposed to lessen these disparities, most recently, a proposal by the current Biden administration to make at least community college free. If however, the main problem holding poor children back from successfully pursuing a college education is not that they cannot pay for it (i.e., the prevalence of binding credit constraints), but rather that they arrive at college age ill-equipped to successfully apply and ultimately graduate from college, policies tackling this problem (such as public funding to improve primary and secondary schools, or transfer payments to poor families) might be more appropriate tools to increase the average and reduce the cross-sectional dispersion and intergenerational persistence of college attendance, earnings, wealth and welfare. This paper seeks to develop a dynamic general equilibrium model of childhood human capital and higher education decisions to assess the positive and normative implications of these policies designed to boost human capital accumulation and college attendance, especially those children from disadvantaged socio-economic backgrounds.

Concretely, we develop a general equilibrium overlapping generations framework with intergenerational links through altruistically-motivated education and wealth transfers, in the spirit of Barro and Becker (1988), and with rich cross-sectional heterogeneity of labor productivity (and thus earnings), human capital, wealth and marital status. Human capital is accumulated at different stages of a child's development, depending on parental resource- and time investments as well as public education funding. Crucially, human capital acquired at earlier stages of child development determines the productivity of all future human capital investments, and the human capital acquired prior to the higher education (college) stage determines both the chances to succeed in college and the expected returns from a college education. Altruistically motivated and rationally forward-looking parents respond to policies affecting the labor market stage of their children's life cycle. At the same time, parents react to education (financing) reforms by adjusting both their own education choices and investments in their children. These interactions between education subsidies targeted at different stages of child and adolescent development and progressive taxation suggest that these policy reforms must be studied jointly. The dynastic modelling framework with intergenerational linkages allows us to evaluate the implications of policy reforms not only for cross-sectional inequality but also for intergenerational earnings- and education mobility.

Our policy analysis starts from an initial stationary equilibrium calibrated to the status quo of the US economy in the 2010's. We then investigate the impact of policy reforms along the transition of the economy towards a final steady state. Along this transition, the government may issue new government debt to finance education policies that in the long run raise human capital and thus the tax base (a fiscal externality), but take time to materialize its full impact. By issuing debt the government may thus smooth out the transitional costs of the policy reforms.

We evaluate a set of once-and-for-all policy reforms that are motivated by the current political discussion and that are comparable in their short-run government expenditure requirements. The first reform we consider is making college education free (motivated by President Biden's proposal to provide for universal free community college) which in our model is implemented by a 100% tertiary education subsidy by the government. This reform is then used as benchmark to determine the size of the other reforms to make those fiscally comparable. The alternative policy reform we consider focuses on human capital accumulation of younger children by increasing public spending for primary or secondary schools. Finally, we study whether there is scope by combining both reforms by characterizing the optimal mix (according to an explicit social welfare function discussed below) of "better schools" and college tuition subsidies, holding the total fiscal cost of the policy reform on impact constant across all interventions considered.

Our findings can be summarized as follows. In terms of aggregates, both education reforms increase the share of a cohort going to college strongly (and roughly to the same degree in the long run), but the overall expansion of human capital is much more pronounced under the pre-college school spending reform. Because of the stronger increase in human capital in the long run, the net present discounted value of government revenues rises more substantially under the school expenditure expansion than under the 100% college subsidy reform. The expansion of the tax base is then larger in the former reform as well, to an extent that it is self-financing, in the sense that the required (permanent) increase in the labor income tax rate to balance the intertemporal government budget is actually negative. Both reforms generate significant long-run welfare gains (in the order of 12-15% of permanent consumption, when measured as consumption equivalent variation of newborn agents (which in turn includes the welfare benefits of their children, given the assumed altruism). Consistent with the more favorable human capital and tax revenue expansions, the welfare gain is larger by about 3 percentage points in the "better schools" reform than in the free college reform in the long run.

Second, the two policy reforms have vastly different distributional consequences. Most crucially, the pre-college expenditure reform also benefits children from households who will not go to college even if they do not have to pay tuition, as is the case under the free college reform.

This in turn has profound consequences for the intergenerational persistence of earnings and educational attainment. Perhaps the most striking contrast between the reforms is the differential impact on the educational attainment of the poorest children, which tend to be children growing up in a household with a single parent and low (less than high school) educational attainment. The free college reform hardly changes the educational attainment of these children, primarily because their accumulated human capital during childhood makes them very unlikely to go to college and succeed there. In contrast, the additional human capital accumulation these children obtain with the school expenditure reform, although insufficient in most cases to push them above the college threshold, strongly increases the chances of these children to at least complete high school, therefore strongly reducing the intergenerational persistence of dropping out of high school.

To isolate the importance of changes in endogenous interest rates and (relative) wages we also conduct a sequence of partial equilibrium exercises in which we hold these endogenous prices as well as the taxes required to balance the intertemporal government budget constant. We show, broadly speaking, that qualitatively, the aggregate and distributional conclusions discussed above also emerge in the absence of equilibrium price adjustments, but the welfare gains of the reforms are larger (as are the difference between the two reforms) in partial equilibrium. The most important general equilibrium effect in both types of reforms stems from the fact that the supply of labor, especially college labor (but also total labor in efficiency units), strongly increases, inducing a decline in the capital-labor ratio and therefore an increase in the real interest rate and a reduction in the real wage per labor efficiency units. Since the college wage premium also falls, relative to the long run BGP the wages of college graduates decline very substantially, whereas the ages of non-college labor mildly increases.

It turns out that for the size of the general welfare gains, the reduction in wages (and especially, the wages of college graduates) is quantitatively most important (as we demonstrate by considering a thought experiment where we hold wages constant but let interest rates adjust). These general equilibrium wage impacts on welfare are broadly negative, but with a nuance. As discussed, the reduction in the capital-labor ratio leads to a decline in the wage per labor efficiency units. However, the very significant increase in the share of college graduates induced by both reforms leads, over time, to a massive reduction in the college wage premium. Consequentially, college wages fall precipitously along the transition (and there is now a large share of individuals making these wages), in turn muting the increase in the college share in general equilibrium relative to partial equilibrium. In contrast, the wages of workers without a college degree actually slightly increase as the relative wage effect slightly dominates the absolute wage effect for this (shrinking) group of individuals or households. Since the policies lead to wage compression within the population, the distributional consequences for ex-ante lifetime utility are positive, partially compensating for the welfare losses from the decline in the absolute wage level. Finally, the (modest) increase in the real interest rate strengthens savings incentives; in the better schools reform it leads to a shift in the wealth distribution to the right over time and a larger capital stock; in the free college reform it mitigates the decline in private wealth accumulation that otherwise would have occurred in partial equilibrium due to the collapse in inter-vivos transfers (since college is now free instead of in need of private funding by parents or loans taken out by students).

Overall, and relative to a world where all factor prices are constant, the long-run welfare gains for the free college reform are 2.6 percentage points lower in general- than in partial equilibrium. The corresponding fall is 4.6 percentage points for the "better schools" reform. Thus, endogenous factor price movements not only reduce the welfare gains from the reforms, but also reduces the gap in the welfare consequences across the two reforms.

1.1 Related Literature

Our paper seeks to connect two broad literatures and exploit that connection for the study of currently proposed education finance and fiscal policy reforms. The first, and perhaps older literature in quantitative macro public finance, is concerned with (optimal) redistributive tax-transfer and education policies; see Benabou (2002), Hanushek et al. (2003) and Bovenberg and Jacobs (2005) for foundational papers. Recent papers in this genre focusing on education (financing) reform include Abbott et al. (2019), Caucutt and Lochner (2020), Stantcheva (2017), Capelle (2020), Fu et al. (2023) and also Athreya et al. (2019), Fogli et al. (2023) as well as our own work, Krueger and Ludwig (2016). An important part of this literature studies the impact of tax- and education policy on intergenerational mobility,[1] see e.g. Holter (2015), Lee and Seshadri (2019), Koeniger and Prat (2018) and Koeniger and Zanella (2022), and a complementary and equally relevant literature studies (optimal) tax-transfer and poverty alleviation policy (transitions), see, e.g., Boar and Midrigan (2022), Dyrda and Pedroni (2023), Daruich and Fernandez (2023), Floden (2001), Ortigueira and Siassi (2023), Guner et al. (2020) and Guner et al. (2021).[2] In contrast to most of this existing literature, this project takes as central tenet that the heterogeneity in initial conditions at labor market entry with respect to human capital and wealth is an endogenous objects that can be affected by education and fiscal policies. Thus, it considers education policies as additional means of redistribution, by reducing education and achievement

[1]Since intergenerational persistence in outcomes is impacted by intergenerational transfers, the empirical literature on these transfers in, e.g., Gale and Scholz (1994), Altonji et al. (1997) and especially Yang and Ripoll (2023) provides important references for the calibration of our the model.

[2]A complementary empirical literature studies the interaction of welfare programs and the education and human capital accumulation of children, see, e.g., Del Boca et al. (2014), Del Boca et al. (2016), National Academies of Sciences and Medicine (2019) and Bailey et al. (2023)

gaps of children from different socio-economic backgrounds and at different stages of the skill formation process. In addition, we seek to contribute to the literature cited above by developing a framework that can distinguish between the incidence of pre-college versus college subsidies while explicitly modelling the complementarity between ability and educational attainment for wages (see Jacobs and Bovenberg (2011) or Stantcheva (2017)) and the dynamic complementarities in child human capital accumulation recently stressed by Cunha et al. (2010).

Therefore, into the above literature we seek to integrate an explicit modelling of life cycle choices with an explicit production function for human capital at different stages of child development. In this regard the proposal builds on a recent literature in empirical microeconomics and quantitative macroeconomics that models child skill formation and human capital accumulation endogenously, see, e.g., Blandin and Herrington (2022), Cunha et al. (2006), Cunha and Heckman (2007), Cunha et al. (2010),Caucutt et al. (2020), Bolt et al. (2023), Eckstein et al. (2019), Daruich (2022), Yum (2023) and our own work, Fuchs-Schündeln et al. (2022), to study the dynamic interactions between parental borrowing constraints and public education spending.[3] On the modelling side we extend this literature by considering the endogenous time allocation choice for both parents between work, leisure and spending time with children of different ages. We also emphasize the importance of general equilibrium effects induced by the policy interventions. On the applied policy side, our main focus lies on the impact of (optimal) policy transitions (permitting government debt) on cross-sectional inequality and intergenerational persistence of economic outcomes, especially those at the lower end of the income and wealth distribution. This in turn requires the explicit model with intergenerational linkages and rich household heterogeneity especially with respect to family marital structure that we provide in this paper. It also highlights the importance of distinguishing the impact of policy reforms in the short run (early in the transition) and in the long run (in the final steady state).

2 The Quantitative Model

2.1 Overview

We employ a general equilibrium overlapping generations (OLG) model in which generations are linked through the intergenerational transmission of innate ability and financial wealth transfers. Parents are altruistic towards their children and can invest their time and monetary resources

[3]The *empirical* literature on the impact of day care- and education spending and financing on education and economic outcomes, see, e.g., Havnes and Mogstad (2011), Abramitzky and Lavy (2014), Jackson et al. (2015), Deming and Walters (2017), Johnson and Jackson (2019), Jackson and Mackevicius (2021), Black et al. (2020), Duncan et al. (2022) and Flood et al. (2022) (as well as the survey by Handel and Hanushek (2022)) will provide key targets for our structural model.

into the human capital accumulation of children when the latter are still living in the parental household. In addition, parents can transfer wealth to children directly when they leave the household. The government collects taxes, runs a PAYGO social security system and finances exogenous government spending and endogenous education spending with taxes and government debt, subject to an intertemporal budget constraints. In general equilibrium the goods-, labor- and asset markets have to clear in every period along a policy-reform induced transition.

Relative to the standard quantitative life cycle literature our model contains three key additional features. First, households have children whose human capital accumulation during the transition from childhood to adolescence is endogenous and depends both on public and private parental inputs. This element of the model is crucial for a study of education policies that differ in the extent to which primary/secondary and tertiary education is altered and fiscal policies that impact the trade-off between market work (including participation) and time investment into children.[4] Second, generations in our model are linked through "brains and bucks", that is, human capital inputs and financial transfers from parents to children. With this model element, parents have endogenous margins of adjustment in direct response to both headline policy reforms. If college will be free, private inter-vivos transfers (and the accumulation of parents assets to make these transfers) will endogenously adjust. When public schools become better, private time and resource inputs can respond as well. Third, modelling both for married households but also single mothers allows us to explicitly account for a group of children that disproportionally grow up in poverty and are least likely to go to college. We now describe the model in greater detail.

2.2 Individual State Variables

In order to meaningfully study the distributional consequences of the proposed policy reforms the model features rich cross-sectional heterogeneity, best described in terms of the individual state variables that characterize households. These are summarized in Table 1, including the range of values these state variables can take. Individuals differ by age j and young households start their independent economic life as singles and with four ex-ante predetermined state variables: gender (either being a woman or a man, $g \in \{wo, ma\}$), the education of their parents s_p (which determines their cost of attending college) initial human capital (h) and initial assets (a). After an individual has taken its own higher education decision s, the highest completed education level also becomes a state variable (and that of the parent ceases to be relevant), and upon labor market entry, the acquired human capital stochastically translates into a discrete-valued fixed

[4] The presence of government transfer- and social assistance programs whose importance varies by family structure renders the explicit modelling of an extensive margin, for both partners in a married household, important, as the recent work by Guner et al. (2012), Bick and Fuchs-Schündeln (2017) and Holter et al. (2023) suggests

Table 1: Individual State Variables

State Var.	Values	Interpretation
j	$j \in \{0, 1, \ldots, J\}$	Model Age
g	$g \in \{wo, ma\}$	Gender
h	$h > 0$	Human Capital
a	$a \geq -\underline{a}(j, s)$	Financial Assets
s, s_p	$s \in \{hsd, hs, cod, co\}$	Higher Education
γ	$\gamma \in \{\gamma_l(s), \gamma_h(s)\}$	Fixed Productivity Component
η	$\eta \in \{\eta_l, \eta_h\}$	Persistent Productivity Shock
q	$q \in \{si, cpl\}$	Marital Status

effect γ with education-specific support.[5] Labor productivity is also impacted by a persistent stochastic component η which is part of the state space. Finally, one period before children are born into the household, the marital status q of a household realizes and becomes a state variable, as a fraction of single households marry ($q = cpl$ for "couples") while the rest remains single ($q = si$). When children are born into household, their human capital h becomes a state variable as well.[6] In terms of notation, for married households the education and labor productivity of both partners are state variables, and the notation s_{-g} and γ_{-g} will be used to denote the state variable of the "other" spouse. We now go on to describe the life cycle decision problems of households, including selected dynamic programming problems of households.

2.3 Demographics, Timing and Economic Decisions

Time is discrete, indexed by t and extends to infinity. In every period t the economy is populated by J overlapping generations indexed by j. Individuals survive from age j to age $j+1$ with probability ϕ_{j+1}. Before retirement survival is certain while from the retirement age j_r onwards survival risk becomes relevant. Assets of households that die at age j are distributed in a lump-sum fashion among all working age households[7]. Transfers from accidental bequests are denoted by $Tr_{t,j}$.

Children are born at age j = 0 (biological age 2; the first two years of child lifecycle are discarded). Parental fertility age is denoted by j_f. The number of children per household (fertility rate) is denoted by $\varsigma(s(wo))$ and is a function of the mother's education level. At this age parents draw an initial child human capital level from a distribution that depends on their education level.

[5] The stochastic mapping from human capital h to the fixed labor productivity γ replaces a continuous state variable (h) with a discrete-valued one (γ) which reduces the state space.

[6] Since at that time fixed productivity γ has replaced parental human capital, and thus there is no scope for confusion between parental and child human capital.

[7] To be more precise, what is redistributed among surviving households are the accidental bequests that remain after the amount needed to finance private college subsidies is deducted.

Children stay in the parental household and accumulate human capital depending on their initial human capital and the time and resource input of their parents as described below. These parental investments are referred to as private human capital investments. When children leave the parental household parents give them (non-negative) inter-vivos transfers b which can be used for consumption financing and/or for covering college expenses.

At model age j_a (biological age 18) a college education decision takes place. Those children who choose college spend one model period for education, the other group starts working directly at age j_a. Dropping out of college takes place stochastically with the dropout shock being realized directly before the college education starts[8]. College dropouts are assumed to have to pay two times smaller tuition costs than college graduates, and they also face a (two times) tighter borrowing limit.

At model age j_a all education groups draw a fixed productivity component $\gamma(s,h)$ which has only two realizations - high and low. The probability of drawing a high realization of the fixed effect is an increasing function of acquired human capital. College students (both those who will graduate and those who will drop out) are assumed to work at high school wages[9].

After education is completed all households enter the labor market. When the labor market entry happens acquired human capital seizes to be a state variable for all education groups. College graduates and college dropouts redraw their fixed productivity component based on a newly obtained higher education level.

During the working life, households make a discrete decision whether to work, and conditional on employment endogenously choose hours worked subject to a time endowment constraint. One period before the fertility age households face an exogenous (education specific) probability of marriage, and depending on the realization of the marriage shock continue living to the next period either as singles or as couples. The marriage age is denoted j_m. Retirement takes place exogenously at the model age j_r. The maximum possible lifespan is J.

All choice variables are summarized in Table 2.

2.4 Human Capital

Human Capital Accumulation during Childhood. In every period during childhood human capital accumulation takes places according to the following production function:

$$h' = g\left(j, h, i^m, i^t, i^g\right), \tag{1}$$

[8]TBC: alternatively, dropping out of college can be modelled as an endogenous decision.

[9]This means that both the aggregate wage level as well as the fixed productivity component are the same as for high school graduate workers.

Table 2: Per Period Decision Variables

Control Var.	Values	Decision Period	Interpretation
c	$c > 0$	$j \geq j_a$	Consumption
ℓ	$\ell \geq 0$	$j \geq j_a$	Hours worked (for couples $\ell(wo)$ and $\ell(ma)$)
a'	$a' \geq -\underline{a}(j,s)$	$j \geq j_a$	Asset Accumulation
i^t	$i^t \geq 0$	$j \in \{j_f, ..., j_f + j_a\}$	Time Investments (for couples $i^t(wo)$ and $i^t(ma)$)
i^m	$i^m \geq 0$	$j \in \{j_f, ..., j_f + j_a\}$	Monetary Investments
b	$b \geq 0$	$j = j_f + j_a$	Monetary Inter-vivos Transfer
s	$s \in \{hsd, hs, cod, co\}$	$j = j_a$	(Higher) Education

Notes: List of decision variables of the economic model.

where i^t and i^m denote parental time and monetary investment, while i^g denotes public (time) investment. Some of the parameters of the human capital production function are age-dependent for calibration purposes to capture differences in the relative importance of inputs at different stages of childhood.

For married households the time investment i^t is a composite of the time inputs of both parents which are assumed to be perfectly substitutable:

$$i^t = i^t(wo) + i^t(ma) \tag{2}$$

where $i^t(wo)$ and $i^t(ma)$ denote the time inputs of a *wo*man and of a *ma*n, respectively.

2.5 Higher Education Decision

After leaving the parental household, the first economic decision of children is a discrete choice whether to attend college.

At the beginning of the college period the college completion shock is realized which together with the fixed productivity component determines the household wages for the rest of the lifcycle. The effective cost of college education and borrowing conditions also depend on the realization of the college completion shock - college dropouts pay smaller tuition costs and face a tighter borrowing limit than college graduates.

2.6 Labor Productivity

The wage of a single household at age j, of gender g with an education level s and with a fixed productivity component realization $\gamma(s)$ is given by:

$$w(s, \gamma(s), g, j) = w(s) \cdot \gamma(s) \cdot \epsilon(s, g, j) \cdot \eta \tag{3}$$

where $w(s)$ is the aggregate wage component, $\gamma(s)$ is a fixed household productivity component, $\epsilon(s,g,j)$ is a deterministic gender- and education-specific productivity profile, and η denotes a potentially persistent productivity shock.

For couples, the household wage is a sum of male and female wages:

$$w(s(wo), s(ma), \gamma(s(wo)), \gamma(s(ma)), j) = \tag{4}$$

$$w(s(wo)) \cdot \gamma(s(wo)) \cdot \epsilon(s(wo), g = wo, j) \cdot \eta + w(s(ma)) \cdot \gamma(s(ma)) \cdot \epsilon(s(ma), g = ma, j) \cdot \eta \tag{5}$$

2.7 Decision Problems

Below household decision problems are stated using a recursive formulation. All variables are expressed in per capita terms and detrended by the rate of technological progress μ.

2.7.1 Children

Children are born at age $j = 0$ and are themselves economically inactive until age $j = j_a - 1$; they stay in the parental household and accumulate human capital depending on their initial human capital and the time and resource input of their parents as described below.

2.7.2 Young Adults and the Education Decision at Age j_a

At model age j_a (biological age 18) children have become young adults and form an independent household with initial state (g, s_p, a, h) given by gender, parental education, financial assets and human capital. Now the tertiary education level is determined, partially by choice and partially by chance. It takes four values, as individuals can be high-school dropouts (hsd), high-school graduates (hs), college dropouts (cod) and college graduates (co). First, high school graduation is exogenous from the perspective of the newly founded household but stochastic: with probability $\pi^{hs}(h)$ (which depends positively on human capital h of the individual and thus is influenced by choices parents took during childhood) the individual obtains a high-school diploma and with complementary probability it becomes a high-school dropout, with continuation lifetime utility $V_t(j_a, si, g, hsd, a, h)$ of an age j_a single si of gender g and assets a as well as human capital h.

A high-school graduate can then choose to attend college. Attending college is costly, both in terms of tuition (which is potentially subsidized by the government and can be financed by student loans) as well as in terms of the opportunity cost of time, and subject to exogenous (but human-capital dependent) drop-out risk: individuals succeed in college only with probability $\pi^{co}(h)$.

Individuals weigh these costs against the benefits of higher wages upon college graduation.[10] The college attendance choice can then be written as

$$s = \begin{cases} hs & \text{if } V_t(j_a, si, g, hs; a, h) \geq V_t(j_a, si, g, ce; s_p, a, h) \\ ce & \text{if } V_t(j_a, si, g, ce; s_p, a, h) > V_t(j_a, si, g, hs, s_p; a, h), \end{cases} \tag{6}$$

where $V_t(j_a, g, ce; s_p, a, h)$ is the pre-dropout college *attendance* value function given by:

$$\begin{aligned} &V_t(j_a, si, g, ce; s_p, a, h) = \\ &\pi^{co}(h) \cdot V_t(j_a, si, g, co; s_p, a, h) + (1 - \pi^{co}(h)) \cdot V_t(j_a, si, g, cod; s_p, a, h). \end{aligned} \tag{7}$$

and the pre-college *decision*, age j_a value function is given by

$$\begin{aligned} &V_t(j_a, si, g, s_p; a, h) = (1 - \pi^{hs}(h)) \cdot V_t(j_a, si, g, hsd; a, h) \\ &\quad + \pi^{hs}(h) \cdot (\max_{s \in \{hs, ce\}} \{V_t(j_a, si, g, hs; a, h), V_t(j_a, si, g, ce, s_p; a, h)\}). \end{aligned} \tag{8}$$

2.7.3 First Period of Working Life / College Period

At the beginning of the first period of independent economic life, realizations of the fixed productivity component and idiosyncratic productivity shocks are drawn. Thus, in the fist period of economic life the decision problem can be split in two subperiods. In the first subperiod, the fixed productivity component and the persistent income shock are drawn:

$$V_t(j_a, si, g, s, s^p, h, a) = \sum_{\gamma} \pi^{\gamma}(s, h) \sum_{\eta} \Pi(\eta) V_t(j_a, q = si, g, s, s^p, h, \gamma, \eta, a)$$

where γ denotes education-specific realizations of the fixed productivity component, and η is the persistent productivity shock realization. The probability of drawing a high fixed effect realization is denoted by $\pi^h(s, h)$ and is a function of acquired human capital. For households that neither enroll in college nor complete it, from this point in time onward acquired human capital seizes to be a state variable and is replaced by the fixed effect $\gamma(s)$. Recall, college students work at high school wages during the college phase, therefore for them γ has to be redrawn upon college completion.

[10] College students (both those who will graduate and those who will drop out) can work-part time at high school wages. Additionally, students experience a utility cost of attending college that depend on their acquired human capital h and on the education of their parents s_p. Finally, college dropouts pay smaller tuition costs and face a tighter borrowing limit than college graduates.

After the fixed effect is drawn, a standard consumption-savings problem with endogenous labor supply is solved. For households that choose not to enroll in college, the decision problem is identical to the one described in the next subsection 2.7.4. For households that complete college, the decision problem is slightly modified because they redraw the fixed productivity component given their newly obtained higher education level, incur psychological and financial costs of attending college, can work only up to maximum $\bar{\ell}^{ce}$ and also are allowed to borrow:

$$V_t(j_a, si, g, s, s^p, \gamma(s < co, h), h, \eta, a) = \max_{c,a',\ell \leq \bar{\ell}^{ce}} \{u(c, \ell) - F(g)_{\ell>0} - p(s, s^p; h) + \beta \sum_{\gamma'} \pi^{\gamma'}(s, h) \sum_{\eta'} \pi(\eta'|\eta) V_{t+1}(j+1, si, g, s, \gamma', \eta', a') \Big\}$$

subject to

$$\begin{aligned} a'(1+\mu) + c(1+\tau^c) + T(y(1-0.5\tau^p)) + \iota(1-\varrho-\varrho^{pr}) &= (a + Tr_{t,j})(1 + r(1-\tau^k)) + y(1-\tau^p) \\ y &= w(s)\gamma(s)\epsilon(s, g, j)\eta\ell \\ a' &\geq -\underline{\mathsf{a}}(s, j) \\ c &\geq 0 \\ \ell &\in [0, \bar{\ell}^{ce}]. \end{aligned}$$

where $p(s, s_p; h)$ is the psychological (utility) cost of attending college, and $\iota(1-\varrho-\varrho^{pr})$ is the tuition cost net of public and private subsidies. $F(g)_{\ell>0}$ denotes a fixed utility cost of working positive hours which depends on the household gender.

Households that enroll in college but drop out solve the same problem as above with the only difference that they are assumed to pay only half of the tuition costs.

2.7.4 Working Life Before Marriage

After completing (or not) their tertiary education single individuals enter the labor market and make labor supply as well as consumption-saving choices (c, a', ℓ), in light of their labor productivity, which is determined by an individual fixed effect γ, a deterministic education-, gender- and age-specific life cycle profile $\epsilon(s, g, j)$ and a persistent stochastic component η. The permanent labor productivity type γ is drawn at the beginning of labor market entry.[11]. With probability $\pi^\gamma(s; h)$ permanent productivity is $\gamma = \gamma_l(s)$ and with complementary productivity it is $\gamma = \gamma_h(s)$. The wage of a single individual is then given by $w(s) \cdot \gamma(s) \cdot \epsilon(s, g, j) \cdot \eta$, where $w(s)$ is the education-specific aggregate wage per efficiency unit of labor.

[11] Since college students can also work part-time, they also draw their fixed effect prior to college entry.

During working life, households make the discrete decision whether to work, and conditional on employment endogenously choose hours worked subject to a time endowment constraint. The decision problem of singles can then be written as

$$V_t(j, si, g, s, \gamma, \eta, a) = \max_{c,a',\ell} \Big\{ u(c, \ell) - F(g)_{\ell>0} + \beta \sum_{\eta'} \pi(\eta'|\eta) V_{t+1}(j+1, si, g, s, \gamma, \eta', a') \Big\} \quad (9)$$

subject to

$$\begin{aligned} a'(1+\mu) + c(1+\tau^c) + T(y(1-0.5\tau^p)) &= (a + Tr_{t,j})(1 + r(1-\tau^k)) + y(1-\tau^p) \\ y &= w(s)\gamma(s)\epsilon(s, g, j)\eta\ell \\ a' &\geq -\underline{a}(s, j) \\ c &\geq 0 \\ \ell &\in [0, \Gamma^{si}]. \end{aligned}$$

where $\underline{a}(s, j)$ is an age- and education-specific borrowing limit, and Γ^{si} denotes the per period time endowment of a single household; $F(g)_{\ell>0}$ denotes a fixed, gender-specific utility cost of working positive hours. The household takes as given aggregate wages and interest rates $(w(s), r)$ as well as the proportional tax rates on consumption, asset income and labor income for social security (τ^c, τ^k, τ^p), the nonlinear labor income tax schedule $T(.)$ and well as the transfers Tr_j. Labor income taxes are levied on labor income net of employer contributions to social security $y(1 - 0.5\tau^p)$.

2.7.5 Marriage

Individuals remain single, until at age j_m (and at that age only) they face an exogenous, education-specific probability $\pi^m(s)$ of marriage. Depending on the realization of the marriage shock individuals continue to live as singles or form a new married household. Since a married household is characterized by the education and wage fixed effect of both spouses as well as their combined financial asset positions (all of which are at least partially the result of endogenous choices), at age $j_m - 1$ a single individual has to form expectations over the type of spouse it might marry (and these expectations have to be confirmed in a rational expectations equilibrium, inducing an additional equilibrium fixed point problem). Recalling that state variables of the spouse of the opposite gender are indexed by $-g$, the decision problem at model age $j_m - 1$, in anticipation of potential marriage next period, is given by:

$$
\begin{aligned}
&V_t(j, si, g, s, \gamma, \eta, a) = \max_{c,a',\ell} \{u(c,\ell) - F(g)_{\ell>0} \\
&+\beta(\pi^m(s) E_{a'^{-g},s^{-g},\gamma^{-g}} \sum_{\eta'(wo)} \Pi(\eta'(wo)) \sum_{\eta'(ma)} \Pi(\eta'(ma)) \times \\
&V_{t+1}(j+1, cpl, s(wo), s(ma), \gamma(s(wo)), \gamma(s(ma)), \eta'(wo), \eta'(ma), a'(wo) + a'(ma)) \\
&\qquad\qquad +(1-\pi^m(s)) \cdot \sum_{\eta'} \pi(\eta'|\eta) V_{t+1}(j+1, g, s, \gamma, \eta', a') \Big\}
\end{aligned}
$$

where $E_{a'^{-g},s^{-g},\gamma^{-g}}(\cdot) = \int (\cdot) d\Phi(j_m - 1, q = si, -g, s, \gamma, \eta; a)$, i.e. the expectation over the characteristics of potential spouses is determined by the cross-sectional measure of the opposite gender households in period $j_m - 1$ and $V_{t+1}(j+1, cpl, s, s_{-g}, \gamma, \gamma_{-g}, \eta'(wo), \eta'(ma), a' + a'_{-g})$ is the continuation value function of the newly formed couple. The maximization problem is subject to the following constraints

$$
\begin{aligned}
a'(1+\mu) + c(1+\tau^c) + T(y(1-0.5\tau^p)) &= (a + Tr_{t,j})(1 + r(1-\tau^k)) + y(1-\tau^p) \\
y &= w(s)\gamma(s)\epsilon(s,g,j)\eta\ell \\
a' &\geq -\underline{a}(s,j) \\
c &\geq 0 \\
\ell &\in [0, \Gamma^{si}].
\end{aligned}
$$

2.7.6 Parenthood and Child Human Capital Accumulation

At age $j_f > j_m$ children enter single women- and married households (single men do not live with children). The number of children per household is a function of the mother's marital status and education level, and is denoted by $\varsigma(q, s(wo))$. All children of a household are assumed to be identical and characterized initially by a level of human capital h that depends on parental education and marriage status (s, q). As long as children are present, parents invest time and resources (i^m, i^t) into the production of new child human capital; we term these *private* human capital investments. For married couples, time investment is the sum of time devoted to their children by both partners, $i^t = i^t(wo) + i^t(ma)$. Finally, when children leave the household, parents can give them non-negative inter-vivos transfers b to finance tertiary education (or their consumption).

In every period during childhood private human capital investments are combined with *public* investment into schooling to transform existing child human capital h into new human capital h' according to the following age-dependent production function $h' = g(j, h, i^m, i^t, i^g)$. For single women, the decision problem during this stage of the life cycle then is

$$
\begin{aligned}
V_t(j, si, wo, s, \gamma, \eta; a, h) &= \max_{c, i^m, i^t, a', h', \ell} \left\{ u\left(c, \ell, i^t\right) - F(wo)_{\ell>0} \right. \\
&+ \left. \beta \sum_{\eta'} \pi(\eta'|\eta) V_{t+1}(j+1, si, wo, s, \gamma, \eta'; a', h') \right\} \qquad (10)
\end{aligned}
$$

$$
\begin{aligned}
a'(1+\mu) + c(1+\tau^c) + \varsigma(si, s) \cdot i^m + T(y(1-0.5\tau^p)) &= (a + Tr_{t,j})(1 + r(1-\tau^k)) + y(1-\tau^p) \\
y &= w(s)\gamma(s)\epsilon(s, g, j)\eta\ell \\
a' &\geq -\underline{a}(j, s), c \geq 0 \\
\ell + \varsigma(si, s) \cdot i^t + \xi(j - j_f + 1, q, s) &\leq \Gamma^{si} \\
h' &= g(j, h, i(i^m, i^t, i^g)),
\end{aligned}
$$

where $\xi(j - j_f + 1, q, s)$ denotes a fixed childcare time requirement which is a function of the age of the child as well of marital status and education of the mother. Since single men are assumed not to have children present in the household, they solve the same maximization problem as in (9).

For couples, participation, hours worked and the time investment of both spouses are choice variables, and thus the dynamic programming problem of the household reads as:

$$
\begin{aligned}
&V_t(j, cpl, s(wo), s(ma), \gamma((wo)), \gamma((ma)), \eta(wo), \eta(ma); a, h) = \\
&\max_{c, i^m, i^t(wo), i^t(ma), a', h', \ell(wo), \ell(ma)} \left\{ u\left(c, \ell(wo), \ell(ma), i^t(wo), i^t(ma)\right) - F(wo)_{\ell(wo)>0} - F(ma)_{\ell(ma)>0} \right. \\
&+\beta \sum_{\eta'(wo)} \pi(\eta'(wo)|\eta(wo)) \sum_{\eta'(ma)} \pi(\eta'(ma)|\eta(ma)) \times \\
&\qquad V_{t+1}(j, cpl, s(wo), s(ma), \gamma(s(wo)), \gamma(s(wo)), \eta'(wo), \eta'(ma); a', h') \}
\end{aligned}
$$

subject to

$$
\begin{aligned}
&a'(1+\mu) + c(1+\tau^c) + \varsigma(s(wo)) \cdot i^m + T^{cpl}(y(1-0.5\tau^p)) = (a + 2 \cdot Tr_{t,j})(1 + r(1-\tau^k)) + y(1-\tau^p) \\
&y = w(s(wo))\gamma(s(ma))\epsilon(s(wo), g = wo, j)\eta(wo)\ell(wo) + w(s(ma))\gamma(s(ma))\epsilon(s(ma), g = ma, j)\eta(ma)\ell(ma) \\
&a' \geq -\underline{a}(j, \max(s(wo), s(ma))) \\
&c \geq 0 \\
&\ell(wo) + \ell(ma) + \varsigma(s(wo)) \cdot (i^t(wo) + i^t(ma)) + \xi(j - j_f + 1, q, s(wo), s(ma)) \leq \Gamma^{cpl} \\
&h' = g(j, h, i(i^m, i^t, i^g)) \\
&i^t = i^t(wo) + i^t(ma),
\end{aligned}
$$

where Γ^{cpl} denotes the couple's time endowment, and $T^{cpl}(.)$ is the labor income tax function it faces.

2.7.7 Children Leaving the Household and Inter-Vivos Transfers

At age $j_f > j_m$ children enter single women- and married households (single men do not live with children). The number of children per household is a function of the mother's marital status and education level, and is denoted by $\varsigma(q, s(wo))$. All children of a household are assumed to be identical and characterized initially by a level of human capital h that depends on parental education and marriage status (s, q). As long as children are present, parents invest time and resources (i^m, i^t) into the production of new child human capital; we term these *private* human capital investments. For married couples, time investment is the sum of time devoted to their children by both partners, $i^t = i^t(wo) + i^t(ma)$. Finally, when children leave the household, parents can give them non-negative inter-vivos transfers b to finance tertiary education (or their consumption).

In every period during childhood private human capital investments are combined with *public* investment into schooling to transform existing child human capital h into new human capital h' according to the following age-dependent production function $h' = g(j, h, i^m, i^t, i^g)$. For single women, the decision problem during this stage of the life cycle then is

$$V_t(j_a + j_f, si, wo, s, \gamma, \eta; a, h) = \max_{c,b,a',\ell} \{u(c, \ell) - F(g)_{\ell>0}$$
$$+\beta \sum_{\eta'} \pi(\eta'|\eta) V_{t+1}(j_a + j_f + 1, si, wo, s, \gamma, \eta'; a')$$
$$+\nu\varsigma(s) E_{g^{ch}} V_t \left(j_a, g^{ch}, s; \frac{b}{1 + r(1 - \tau^k)}, h\right)\right\},$$

where $V_t\left(j_a, g^{ch}, s; \frac{b}{1+r(1-\tau^k)}, h\right)$ denotes the pre-education decision value function of children. Maximization is subject to

$$a'(1 + \mu) + c(1 + \tau^c) + \varsigma(s) \cdot b + T(y(1 - 0.5\tau^p)) = (a + Tr_{t,j})(1 + r(1 - \tau^k)) + y(1 - \tau^p)$$
$$y = w(s)\gamma(s)\epsilon(s, g, j)\eta\ell$$
$$a' \geq -\underline{a}(s, j)$$
$$c \geq 0$$
$$\ell \in [0, \Gamma^{si}].$$

After children have left the household, parental households continue solving a consumption-savings problem with endogenous labor supply they reach retirement:

$$V_t(j, si, wo, s, \gamma, \eta, a) = \max_{c,a',\ell} \Big\{ u(c,\ell) - F(g)_{\ell>0} \\ + \beta \sum_{\eta'} \pi(\eta'|\eta) V_{t+1}(j+1, si, wo, s, \gamma, \eta', a') \Big\}$$

subject to

$$\begin{aligned} a'(1+\mu) + c(1+\tau^c) + T(y(1-0.5\tau^p)) &= (a + Tr_{t,j})(1 + r(1-\tau^k)) + y(1-\tau^p) \\ y &= w(s)\gamma(s)\epsilon(s,g,j)\eta\ell \\ a' &\geq -\underline{a}(s,j) \\ c &\geq 0 \\ \ell &\in [0, \Gamma^{si}]. \end{aligned}$$

2.7.8 Retirement and Death

In retirement, that is after reaching the model age j_r, households solve a standard consumption-saving problem, receive social security benefits $pen(s, \gamma, \eta(j_r - 1))$ and face mortality risk (until they die for sure at maximal lifetime J). The problem reads as

$$\begin{aligned} V_t(j, si, g, s, \gamma, \eta; a) &= \max_{c,a' \geq 0} \{ u(c) + \beta\phi(j) V_{t+1}(j+1, si, g, s, \gamma, \eta; a') \} \qquad \text{s.t.} \\ a'(1+\mu) + c(1+\tau^c) &= (a + Tr_{t,j})(1 + r(1-\tau^k)) + pen(s, \gamma, \eta) \end{aligned}$$

where $pen(s, \gamma, \eta(j_r - 1))$ is retirement income which depends on education-specific wages $w(s)$, the persistent shock realization in the last working period[12] before retirement η, the education level s and the fixed productivity component γ.

We now embed this life cycle model with altruistically linked generations into a general equilibrium neoclassical production economy with a government that sets potentially time-varying tax-transfer and education policies. Time is indexed by t.

[12]This construction allows us to capture the progressivity embedded in the actual US social security benefit formula without carrying around another continuous state variable during working age.

2.8 Production

A representative firm employs capital K_t and aggregate labor L_t to produce the final output good Y_t employing a Cobb-Douglas production function

$$Y_t = K_t^{\alpha}(\Upsilon_t L_t)^{1-\alpha},$$

where α determines the elasticity of output with respect to capital and Υ_t is the technology level growing at the exogenous rate μ, $\Upsilon_{t+1} = (1+\mu)\Upsilon_t$. In order to permit the possibility that a policy-induced change in the share of college graduates changes their relative wages we assume that non-college labor (including college dropouts) and college labor (i.e., college graduates) are imperfectly substitutable in production. Aggregate labor L_t at time t is given by

$$L_t = (L_{t,nc}^{\rho} + L_{t,co}^{\rho})^{\frac{1}{\rho}}, \tag{11}$$

where ρ governs the elasticity of substitution between college $L_{t,co}$ and non-college labor efficiency units and $L_{t,nc} = L_{t,hsd} + L_{t,hs} + L_{t,cod}$ are the labor efficiency units jointly supplied by high-school dropouts, high-school graduates and college dropouts supplied in period t.

2.9 Government

The government administers a progressive labor income tax code, pays transfers to households and collects linear taxes on consumption and capital income. Aggregate labor income tax revenues net of transfers are denoted by T_t. In addition, the government spends $\alpha_j i^g$ per child on primary and secondary school education. The age profile α_j permits us to differentiate between the cost of primary and secondary school and i^g measures the scale of public education spending, and will be one key policy choice by the government. Total spending on primary and secondary public schools is denoted by E_t. The government also subsidizes tertiary education, with the share ϱ of tuition covered by the government; ϱ is the second crucial policy choice variable, and a choice of $\varrho = 1$ represents free college. We denote by E_t^{CL} the aggregate cost of college subsidies.

In addition to the endogenous streams of education expenditures (E_t, E_t^{CL}) for primary, secondary and tertiary education the government also needs to finance an exogenous stream of non-education related expenditures G_t. To do so, the government raises revenues from taxing labor- and capital income as well as consumption, and from issuing government debt B_t. The period t flow government budget constraint then reads as

$$E_t + E_t^{CL} + G_t + (1+r_t)B_t = (1+\mu)(1+n)B_{t+1} + T_t + \tau_{c,t}C_t + \tau_{k,t}r_t(K_t + B_t) \tag{12}$$

The initial stock of government debt B_0 is an exogenously given initial condition (as is the initial aggregate capital stock K_0). Finally, the government also runs a pure pay-as-you-go social security system whose budget equates payroll taxes (with tax rate τ^p) to all pension benefits paid out according to the benefit formula $pen(s, \gamma, \eta)$.

3 Equilibrium Definition and Computation

The key equilibrium object in our model is the cross-sectional measure Φ_t over household characteristics[13] $(j, q, g, s, \gamma, \eta, a, h)$. For each time period t and age j we normalize the total measure $\Phi_t(j, \cdot)$ to 1 and denote by N_j the (time-invariant) size of age cohort j.

$$\int d\Phi_t(j, si, g, s, \gamma, \eta, a, h) + \int d\Phi_t(j, cpl, s(ma), \gamma(wo), \gamma(ma), \eta(wo), \eta(ma), a, h) = 1 \quad (13)$$

In order to clarify the distinction between the partial- and the general equilibrium versions of the model it is necessary to give a somewhat formal definition of equilibrium.

3.1 Equilibrium Definition

For given initial physical capital stock and government debt (K_0, B_0) and initial cross-sectional distributions of singles $\{\Phi_0(j, si, \cdot)\}_{j_a}^J$ and couples $\{\Phi_0(j, cpl, \cdot)\}_{j_m}^J$ a competitive equilibrium is given by sequences of household value and policy functions (for consumption, assets, labor supply, child human capital investments and bequests), aggregate capital and labor inputs, tax and transfer policies and government debt levels, aggregate prices, accidental bequests as well as household measures such that

1. In each period, household value and policy functions solve the household optimization problems, given factor prices, government policies and accidental bequests.

2. Defining the ratio capital to efficiency units of labor as $k_t = K_t/(\Upsilon_t L_t)$, and denoting the exogenous depreciation rate of capital by δ, factor prices for capital and college- as well as non-college labor per efficiency unit satisfy

$$r_t = \alpha k_t^{\alpha-1} - \delta \quad (14)$$

$$w_{co,t} = (1-\alpha) k_t^{\alpha} \left(\frac{L_t}{L_{t,co}}\right)^{1-\rho} \quad \text{and} \quad w_{nc,t} = (1-\alpha) k_t^{\alpha} \left(\frac{L_t}{L_{t,nc}}\right)^{1-\rho} \quad (15)$$

[13] It is understood that, depending on the age j of the household as well as its marital status q, the household state space changes; for example, for couples it includes the education and fixed effect of both partners.

3. Government budget constraint (43) and social security system budget constraint holds $\forall t$

$$\tau_t^p(w_{co,t}L_{co,t} + w_{nc,t}L_{nc,t}) = \sum_{j=j_r}^{J} N_j \int pen_t(s,\gamma,\eta) d\Phi_t \tag{16}$$

4. Markets clear in all periods t:

$$L_{co,t} = \sum_{j_a}^{j_r-1} N_j \int \gamma\epsilon(co,g,j)\eta\ell_t(j,co,\cdot)d\Phi_t \tag{17}$$

$$L_{nc,t} = L_{t,hsd} + L_{t,hs} + L_{t,cod} = \sum_{j_a}^{j_r-1} N_j \sum_{s\in\{hsd,hs,cod\}} \int \gamma\epsilon(s,g,j)\eta\ell_t(j,s,\cdot)d\Phi_t \tag{18}$$

$$K_{t+1} + B_{t+1} = \sum N_{j=ja}^{J} \int a_t'(j,\cdot)d\Phi_t(j,\cdot) \tag{19}$$

$$C_t + K_{t+1} + CE_t + E_t + G_t = K_t^{\alpha}(L_t)^{1-\alpha} + (1-\delta)K_t \tag{20}$$

where L_t was defined in (11) and CE_t are aggregate private education expenditures.

5. Marriage market equilibrium: for each men education type $s(ma)$, the share of women married to this type is equal to the share of married men with $s(ma)$. The same is true for women. [14]

6. The total accidental bequests received by the working age population in period $t+1$ are equal to the total assets of the dead in period t net of private college subsidies

$$\sum N_{t+1,j}{}_{j=j_a}^{j_r-1} Tr_{t+1,j} = \sum N_{t,j}{}_{j=j_r}^{J} \int (1-\phi_j)a_t'(j,\cdot)d\Phi_t(j,\cdot) \tag{21}$$

7. The cross-sectional measures of households evolve according to the laws of motion induced by exogenous population dynamics, the exogenous Markov processes for idiosyncratic labor productivity, the exogenous transitory shocks law of motion, endogenous asset and child human capital (when children are present in the household) accumulation, higher education and inter-vivos transfer decisions, both at the age of marriage and at all other ages.

[14] Formally,

$$\sum_{s(wo)} \pi^m(s(wo)|s(ma))\Phi_t(j_m-1,si,wo,s(wo),\cdot) = \sum_{s(wo)} \pi^m(s(ma)|s(wo))\Phi_t(j_m-1,si,ma,s(ma),\cdot)$$

$$\sum_{s(ma)} \pi^m(s(ma)|s(wo))\Phi_t(j_m-1,si,ma,s(ma),\cdot) = \sum_{s(ma)} \pi^m(s(wo)|s(ma))\Phi_t(j_m-1,si,wo,s(wo),\cdot)$$

8. The initial measure of newly formed households $\Phi_t(j_a, si, \cdot)$ at age j_a is consistent with inter-vivos transfers and human capital investment decisions of parents and the measure of economic newborns at age j_a after the higher education choice is made.

9. At age $j_m - 1$ prior to marriage expectations of singles about characteristics of future spouses are consistent with the cross-sectional distribution of the opposite gender at age $j_m - 1$.

3.2 Solution Algorithm

We propose to solve for (optimal) policy transitions in a model characterized by non-convex household maximization problems involving discrete and continuous decision variables as well as a sizeable individual state space, and in which there are two nested fixed-point problems even in partial equilibrium, one emerging from the intergenerational linkages (the value function of children enters lifetime utility of their altruistic parents) and one from the marriage market equilibrium (types of pre-marriage singles are endogenous and have to match and conform to household expectations). The solution of market clearing prices in steady state and along the transition path is then relatively standard; here we focus on the more novel fixed point problems in steady state.[15]

Modeling of marriage requires that the marriage market clears which results in a fixed point problem in distributions. Assuming rational expectations implies that before the marriage period expectation of assets and productivity of a future spouse should be consistent with the cross-sectional distribution of assets and productivity of the opposite gender (for a given education level), $\Phi(j_{m-1}, si, g, s, a, \gamma(s))$. Recall that due to explicitly modelled intergenerational altruism, the initial measure of economic newborns $\Phi(j_a, si, \cdot)$ must be consistent with inter-vivos transfers and human capital investment decisions of parents. This implies a second fixed point problem in distributions. Additionally, the value function of the child generation at age j_a should be consistent with the value function of the parental generation at age j_a which turns the finite horizon life cycle problem of each generation into an infinite horizon problem over time. Given that each iteration of the latter fixed point problem is affected by $\Phi(j_{m-1}, si, g, s, a, \gamma(s))$ the three fixed point problems (one in value functions and two in measures) have to be solved jointly.

Thus, aggregation of the model requires solving the two fixed point problems in distributions one of which interacts with the household problem solution because the cross-sectional distribution

[15]The algorithm for the household problem is a combination of the discrete-continuous endogenous grid method described in Iskhakov et al. (2017)), embedded in a value function iteration algorithm that draws on Druedahl (2021).

at model age j_m determines the continuation value before the marriage period. To deal with this multi-layer fixed point problem, we propose the following algorithm:

Step 1: Guess distribution of assets, fixed productivity and education for both genders at the end of period j_{m-1} (for a given skill level s), $\Phi(j_m - 1, si, g, s, a, \gamma(s))$

Step 2: For given $\Phi(j_{m-1}, si, g, s, a, \gamma(s))$, solve for intergenerational RE equilibrium:

  2.1: Solve fixed point problem in value functions (guess $V(j_a, si, \cdot)$, iterate till convergence)

  2.1: Solve fixed point problem in distributions (guess $\Phi(j_a, si, \cdot)$ iterate till convergence)

Step 3: If $||\Phi(j_{m-1}, si, g, s, a, \gamma(s))^{\text{update}} - \Phi(j_{m-1}, si, g, s, a, \gamma(s))^{\text{guess}}|| < \epsilon$, EXIT, else go back to Step 1 and continue until convergence.

---

4 Calibration

The model is calibrated to US aggregate and cross-sectional data, using a standard two-stage procedure in which a subset of the parameters is chosen outside the model an based on values in the literature, and a second set of parameters is calibrated inside the model.

Specifically, while most demographic, aggregate technology and fiscal policy parameters as well as individual labor productivity parameters are set exogenously or directly estimated from the data, the key parameters governing preferences and the child human capital production function are calibrated internally so that the initial steady state general equilibrium of the model is consistent with the (child) age profile of parental time and resources investments, average hours worked, labor force participation, the cross-sectional wage- and education distributions, as well as the level of government spending and government debt. Table 3 summarizes the subset of parameters calibrated exogenously outside the model, and Table 4 provides an overview of the second stage parameters that are calibrated endogenously within the model. We now discuss the rationale for our choices in detail.

4.1 Demographics

The population growth rate n is assumed to be 1% which is the average of the US annual population growth rate values in 2000s. The number of children per mother (fertility rate) differs by education level and is ca. 15% higher for households without a college degree, in accordance with the five most recent PSID waves.

Table 3: First Stage Calibration Parameters

Parameter	Interpretation	Value	Source (data/lit)
	Population		
$j = 0$	Age at economic birth (age 2)	0	
j_a	Age at beginning of econ life (age 18)	4	
j_c	Age at finishing college (age 24)	5	
j_f	Fertility Age (age 32)	7	
j_r	Retirement Age (age 66)	16	
J	Max. Lifetime (age bin 98-101)	24	
$\{\phi_j\}$	Survival Probabilities	see main text	Life Tables SSA
n	Population Growth Rate	1%	
$\frac{\varsigma(s<co)}{\varsigma(s=co)}$	Fertility Education Gradient	1.15	PSID 2011-2019
$\pi^m(s)$	Marriage probability	0.51	PSID 2011 - 2019
	Preferences		
σ	Relative risk aversion parameter	1	
ψ	Frisch elasticity	0.6	
	Labor Productivity		
$\{\epsilon(s, g, j\}$	Age Profile	see main text	PSID 1968-2012
$[\eta_l, \eta_h]$	States of Markov process	[0.6725, 1.3275]	PSID 1968-2012
π_{hl}	Transition probability of Markov process	0.1765	PSID 1968-2012
	Ability/Human Capital and Education		
ι	College tuition costs (annual, net of grans and subsidies)	15,500$	NCES (average 2000-2019)
$\underline{a}(j \in [j_a], co)$	College borrowing limit	45,590$	Krueger and Ludwig (2016)
σ^h	Elast of subst b/w human capital and CES inv. aggr.	1	Cunha et al. (2010)
σ^g	Elast of subst b/w public inv. and CES aggr. of private inv.	2.43	Kotera and Seshadri (2017)
σ^m	Elast of subst b/w monetary and time inv.	1	Lee and Seshadri (2019)
$\Phi(h(j = 0)\|s_p)$	Innate ability dist-n of children by parental education	see main text	PSID CDS I
$\underline{h}_0$	Normalization parameter of initial dist-n of initial ability	0.1248	PSID CDS I-III
	Baseline Government policy		
ϱ	Public subsidy of college education	38.8%	Krueger and Ludwig (2016)
ϱ^{pr}	Private subsidy of college education	16.6%	Krueger and Ludwig (2016)
i_j^g	Public high school education spending	$\approx 14,000$$	NCES (2000-2018)
τ_c	Consumption Tax Rate	5.0%	legislation
τ_k	Capital Income Tax Rate	36%	Trabandt and Uhlig (2011)
ξ	Labor Income Tax Progressivity	0.18	Heathcote et al. (2017)
ω	Income of non-working households	20.2% of average earnings	CEX 2001-2007 (see Holter et al. (2023))
τ^p	Soc Sec Payroll Tax	12.4%	legislation
G/Y	Government consumption to GDP	13.8%	current value

Notes: First stage parameters calibrated exogenously by reference to other studies and data.

Table 4: Second Stage Calibration Parameters

Parameter	Interpretation	Value
	Preferences	
β	Time discount rate (target: interest rate)	0.9949
ν	Altruism parameter (target: average IVT transfer per child)	0.5944
ϕ	Weight on hours disutility[16] (target: average hours per hh)	17.32
F	Fixed cost of working positive hours (target: employment rate)	0.06
	Labor Productivity	
$\rho_0(s)$	Normalization parameter (target: $\mathbb{E}\gamma(s,h) = 1$)	$[1.0115, 1.0115, 0.9752, 0.9110]$
	Human Capital and Education	
κ	Utility weight on time inv.(target: average time inv.)	0.4354
κ_j^h	Share of human capital (target: slope of time inv. and i^g-elasticity)	cf. Figures in main text TBC
κ_j^m	Share of monetary input (target: average monetary inv. & slope)	cf. Figures in main text TBC
$\kappa_j^g = \bar{\kappa}^g, j > 0$	Share of government input for ages 6 and older (target: test score dispersion at ages 17-19 in PSID CDS III)	0.75
κ_0^g	Share of government input for age bin 2-6 (target: average time inv. age bin 2-6)	0.4437
$\bar{A}$	Investment scale parameter (target: normalization of average HK at age j_a)	1.1989
$\varrho(s^p < co)$	Psychological (utility) costs $s = co, s^p < co$ (target: fraction of group $s = co$)	3.1272
$\varrho(s^p = co)$	Psychological (utility) costs $s = co, s^p = co$ (target: conditional fraction of group $s = co$)	1.0096
	Government policy	
τ	Level parameter of HSV tax function (target: balance intertemporal government budget - match B/Y of 100%)	0.21
ρ^p	Pension replacement rate (target: balance per period social security budget)	0.1893

Notes: Second stage parameters calibrated endogenously by targeting selected data moments.

4.2 Technology

The capital share parameter α is set to $1/3$, a standard value in the literature, and the annual physical capital depreciation rate equals 5%. The rate of technological progress (and thus the long-run growth rate of per-capita income) g equals 1%. Finally, the elasticity of substitution between skilled (college) and non-skilled (high school dropout, graduate and college dropout) labor is set to 3.3, following the estimate of Abbott et al. (2019).

4.3 Preferences

For single households, the per period utility function takes the following functional form:

$$u(c,n) = \frac{c^{1-\sigma}}{1-\sigma} - \phi \frac{\ell^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} \tag{22}$$

where $\sigma = 1$, i.e. assume logarithmic utility[17]. Parameter ψ that can directly be interpreted as the Frisch elasticity of labor supply is set to 0.6 following Kindermann and Krueger (2014)[18]. Finally, parameter ϕ is calibrated endogenously to match the average hours worked of $1/3$ of the time endowment.

Households composed of couples experience disutility from hours worked of both partners:

$$u(c,n) = \frac{c^{1-\sigma}}{1-\sigma} - \phi \frac{\ell(wo)^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} - \phi \frac{\ell(ma)^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}}. \tag{23}$$

The term capturing fixed costs of working positive hours $F_{\ell>0}$ is also calibrated endogenously to match the average share of non-participating and unemployed households of 25%.

During the model periods when children live in the parental household also time spent with children affects parental utility. We assume that the disutility from time with children enters the utility function of parents in an additively separable manner[19]:

$$u(c,n) = \frac{c^{1-\theta}}{1-\theta} - \phi \frac{\ell^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} - \kappa \frac{\varsigma \cdot i^{t^{1+\frac{1}{\psi}}}}{1+\frac{1}{\psi}} \tag{24}$$

[17] Given the logarithmic utility assumption, the child equivalence scale parameter is irrelevant for the household problem and for brevity considerations is omitted.

[18] As Kindermann and Krueger (2014) point out this value is based on the average of estimates for men and for women.

[19] Bastian and Lochner (2020) based on females responses to EITC expansions point out that mothers increase their time with children not at the cost of hours worked but rather via reallocating their leisure time.

where κ is calibrated to match the average household time investment into children (per week per child), and $\varsigma(s)$ is the average number of children per household.

For couple households, accordingly, there are additional terms capturing disutility from hours worked and time with children of the second partner:

$$u(c,n) = \frac{c^{1-\theta}}{1-\theta} - \phi\frac{\ell(wo)^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} - \phi\frac{\ell(ma)^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} - \kappa\frac{\varsigma(s(wo))\cdot i^t(wo)^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} - \kappa\frac{\varsigma(s(wo))\cdot i^t(ma)^{1+\frac{1}{\psi}}}{1+\frac{1}{\psi}} \quad (25)$$

When children attend college, they experience utility (psychological) costs determined by the following cost function

$$p(s,s_p;h) = \varrho(s,s_p) + \frac{1}{h}$$

where $\varrho(s,s_p)$ is a calibration parameter which depends on parental education and is 50% smaller for college dropouts than for college graduates, due to the assumed lesser time these individuals spend in college. Specifically, $\varrho(s=co,s_p<co)$ is calibrated to match the average college enrollment rate while $\varrho(s=co,s_p=co)$ is chosen such that the college enrollment rate conditional on parents being college graduates equals 92% (PSID 2011-2019). Observe that the psychological cost specification above implies that the utility costs are monotonically decreasing and convex in the acquired human capital h.

Households discount utility at rate β which is chosen such that in general equilibrium the implied interest rate equals 3.0%. Utility of future generations is discounted at rate ν which governs the degree of parental altruism. Parameter ν is chosen so that average per child inter-vivos transfer is ca. 61,200$, as implied by the Rosters and Transfers Module 2013 of PSID (based on monetary transfers from parents to children until age 26).

4.4 Human Capital Production Function

Initial Child Human Capital In the model innate human capital (at biological age 2) depends on parental education and marital status and for given parental background is exogenously given. The dependency of innate child ability on parental background is disciplined using child test score data from the Child Development Supplement (CDS) to the PSID.

Human Capital Production Function At ages $j_0,\ldots,j_a-1$, children then receive parents' human capital investments through money and time $i^m(j), i^t(j)$ and governmental (schooling) input i^g, respectively. Human capital is accumulated according to a multi-layer human capital production function with imperfectly substitutable inputs:

$$h'(j) = \left(\kappa_j^h h^{1-\frac{1}{\sigma^h}} + (1-\kappa_j^h) i(j)^{1-\frac{1}{\sigma^h}}\right)^{\frac{1}{1-\frac{1}{\sigma^h}}} \tag{26a}$$

$$i(j) = \bar{A}\left(\tilde{\kappa}_j^g \left(i^g\right)^{1-\frac{1}{\sigma^g}} + (1-\tilde{\kappa}_j^g)\left(i^p(j)\right)^{1-\frac{1}{\sigma^g}}\right)^{\frac{1}{1-\frac{1}{\sigma^g}}} \tag{26b}$$

$$i^p(j) = \left(\tilde{\kappa}_j^m \left(i^m(j)\right)^{1-\frac{1}{\sigma^m}} + (1-\tilde{\kappa}_j^m)\left(i^t(j)\right)^{1-\frac{1}{\sigma^m}}\right)^{\frac{1}{1-\frac{1}{\sigma^m}}}. \tag{26c}$$

The production function features partially age dependent parameters for calibration purposes - to reflect relative differences in importance of different inputs at different stages of childhood. All inputs are divided by their respective unconditional means which allows to achieve unit independence (see Cantore and Levine (2012)). This normalization is accounted for by adjusting the weight parameters $\tilde{\kappa}_j^g$ and $\tilde{\kappa}_j^m$, respectively - see Appendix B.6 for details.

In the outermost nest of the production function, existing human capital h is combined with aggregate investment $i(j)$ at age j. The substitution elasticity σ^h is set exogenously to 1 for all ages (implying a Cobb-Douglas specification). The age profile for the weight parameter $\kappa^h(j)$ is calibrated to match the age profile of (per child) parental time investment in the data. The average κ^h, in turn, is chosen such that the average short-run college enrollment elasticity with respect to high school spending generosity matches the midpoint of the range of empirical estimates reviewed in the meta-study Jackson and Mackevicius (2023) on this topic.

In the second nest of the production function public and private inputs $(i^g, i^p(j))$ are combined, with the substitution elasticity between the two inputs being denoted by σ^g and the age-specific weight parameters $\tilde{\kappa}^g(j)$. The substitution elasticity is set exogenously to 2.43 using the estimate provided in Kotera and Seshadri (2017). For kindergarten ages, i.e. age bin 2-6, the weight parameter is calibrated endogenously to match average parental time investment at that age. For other ages, the weight parameter is also calibrated endogenously such that the inequality in acquired human capital by family background is close to the dispersion of test scores in CDS-PSID at ages 17-19. $\bar{A}$ is a normalization parameter which is chosen such that average acquired human capital at age 18 is equal to 1.

Finally, in the innermost nest parental time and resource inputs $(i^t(j), i^m(j))$ are combined, with a substitution elasticity that is denoted by σ^m and the age-dependent (adjusted to achieve unit invariance) weight parameter $\tilde{\kappa}^m(j)$. The substitution elasticity σ^m is fixed exogenously at the value of 1 using the estimate provided in Lee and Seshadri (2019) whereas the weight parameter $\tilde{\kappa}^m(j)$ is calibrated endogenously to match the mean and the age profile of the parental monetary input.

4.5 College Dropout

The probability of finishing college takes the following functional form:

$$\pi^c(h) = 1 - \exp(-\lambda^c h) \tag{27}$$

where λ^c is a parameter calibrated endogenously to match the average share of college dropouts in PSID data[20] of 29%. Observe that for $\lambda^c > 0$ this functional form specification implies that the probability of finishing college is increasing in acquired human capital.

4.6 College Tuition Costs & Borrowing Constraint of Students

Based on NCES statistics, the net tuition cost ι (tuition, fees, room and board rates charged for full-time students in degree-granting postsecondary public institutions) for one year of college in constant 2010 dollars has been on average 15,500$ during the time period 2000-2019. Following Krueger and Ludwig (2016), the maximum amount of publicly provided students loans per year is given by $11,397\$$, which is the borrowing limit for college students in the model. For college dropouts, we assume that the borrowing limit is twice as tight as for college graduates. For all ages after the college period (i.e. for all $j > j_a$) we let

$$\underline{\mathsf{a}}(j, s > hs) = \underline{\mathsf{a}}(j-1, s > hs)(1+r) - rp$$

and compute rp such that the terminal condition $\underline{\mathsf{a}}(j_r, s) = 0$ is met.

4.7 Education Spending

The government spends on schooling for children and pays the college subsidy for college students. According to NCES statistics, average per student spending on public schools is ca. $14,000. The public college subsidy is set to 38.8% of average gross tuition costs, as in Krueger and Ludwig (2016). Additionally, we also explicitly model private subsidies that are paid from accidental bequests and constitute 16.6% of the gross tuition cost in the baseline (see Krueger and Ludwig (2016)).

4.8 Productivity

We use PSID data to regress by education of the household head log wages measured at the household level on a cubic in age of the household head, time dummies, family size, a dummy

[20] Education shares are based on the five recent waves of PSID: 2011, 2013, 2015, 2017 and 2019.

for marital status, and person fixed effects. Predicting the age polynomial gives our estimates of $\epsilon(s,g,j)$. We next compute log residuals and estimate moments of the earnings process by GMM and pool those across education categories and marital status. We assume a standard process of the log residuals according to a permanent and transitory shock specification, i.e., we decompose log residual wages $\ln(\mathrm{y}_t)$ as

$$\ln(\mathrm{y}_t) = \ln(\mathrm{z_t}) + \ln(\varepsilon_t)$$
$$\ln(\mathrm{z_t}) = \rho \ln(\mathrm{z_{t-1}}) + \ln(\nu_t)$$

where $\varepsilon_t \sim_{i.i.d} \mathcal{D}_\varepsilon(0,\sigma_\varepsilon^2)$, $\nu_t \sim_{i.i.d} \mathcal{D}_\nu(0,\sigma_\nu^2)$ for density functions $\mathcal{D}$, and estimate this process pooled across education and marital status. To approximate this process in our model, we translate it into a 2-state Markov process targeting the conditional variance of $\mathrm{y_t}$, conditional on $\mathrm{y_{t-4}}$, $(1+\rho^2+\rho^4+\rho^6)\sigma_\nu^2$ (accounting for the four year frequency of the model). The estimates and the moments of the approximation are reported in Table 5.

Table 5: Stochastic Wage Process

	Estimates			Markov Chain	
Parameter	ρ	σ_ν^2	σ_ε^2	$\pi_{hh} = \pi_{ll}$	$[\eta_l, \eta_h]$
Estimate	0.9559	0.0168	0.0566	0.8235	$[0.6725, 1.3275]$

Notes: This table contains the estimated parameters of the residual log wage process.

Acquired Human Capital and Wages The mapping of human capital into a fixed productivity component is probabilistic. The fixed effect $\gamma(s)$ can take two values, $\gamma^h(s)$ (high) and $\gamma^l(s)$ (low), respectively, for each education group. The probability of drawing a high realization $\gamma^h(s)$ is given by

$$\pi^h(h) = 1 - \exp(h) \tag{28}$$

where h is child acquired human capital at age 18.

Education-specific permanent productivity parameters $\gamma^h(s)$ and $\gamma^l(s)$ are calibrated endogenously to ensure that for each of education group the average $\gamma(s)$ is equal to one[21], i.e.

$$\int \left(\pi^h(s,h)\gamma^h(s) + \left(1-\pi^h(s,h)\right)\gamma^l(s)\right)\Phi(dh,s) = 1. \tag{29}$$

[21] This ensures that the skill premia are matched.

The education-specific spreads $\Delta^{\gamma}(s)$ between $\gamma^{h}(s)$ and $\gamma^{l}(s)$ are calibrated as follows. For high school dropouts and high school graduates that serve as a reference group $\Delta^{\gamma}(s = hsd) = \Delta^{\gamma}(s = hs)$ is set such that the average variance of log wages equals 0.45 as implied by PSID 2011-2019. For the other two education groups, $\Delta^{\gamma}(s)$ parameters are scaled relative to $\Delta^{\gamma}(s < sco)$ such that the ratios of human capital gradients of (lifetime) wages of college graduates and the reference group, on the one hand, and college dropouts and the reference group, on other hand, estimated with NLSY79 data (in expectation, i.e. from an ex ante perspective) are matched. Specifically, estimates of education-specific human capital gradients $\hat{\rho}(s)$ are obtained by running the following regressions:

$$\ln(\omega(s)) = \rho(s) \cdot \frac{e}{\bar{e}} + v(s),$$

where $\omega(s)$ denotes age-free education-specific wages and e measures test scores of the Armed Forces Qualification Test (AFQT) which are normalized by their mean $\bar{e}$. Finally, $v(s)$ is an education group specific error term.

Table 6 shows the resulting estimates $\hat{\rho}(s)$. The estimated ability (human capital) gradient is strictly increasing in education reflecting a pronounced complementarity between ability (human capital) and education.

Table 6: Ability Gradient by Education Level

Education Level	Ability Gradient
(HS- & HS)	0.4248 (0.0481)
(CL-)	0.5786 (0.0245)
(CL & CL+)	0.7298 (0.0670)

Notes: Estimated ability gradient $\hat{\rho}(s)$, using NLSY79 as provided in replication files for Abbott et al. (2019). Standard errors in parentheses.

For $s > hs$, $\Delta^{\gamma}(sco)$ and $\Delta^{\gamma}(co)$ parameters are set such that

$$\frac{\int \left(\frac{\partial \left[\pi^{h}(s,h)\gamma^{h}(s) + \left(1-\pi^{h}(s,h)\right)\gamma^{l}(s) \right]}{\partial h} \right) \Phi(dh, s)}{\int \left(\frac{\partial \left[\pi^{h}(s<co,h)\gamma^{h}(s<co) + \left(1-\pi^{h}(s<co,h)\right)\gamma^{l}(s<co) \right]}{\partial h} \right) \Phi(dh, s<co)} = \frac{\hat{\rho}(s)}{\hat{\rho}(s<co)}.$$

Thus, for given acquired human capital distribution, the education-specific parameters $\gamma^{h}(s)$ and $\gamma^{l}(s)$ jointly determine the dispersion of wages as well as the degree of complementarity between human capital and education in wages. In other words, from an ex ante perspective these parameters determine the steepness of the expected college wage premium in human capital (in expectation), and from an ex post perspective they drive the realized dispersion of wages. The difference between wage dispersion of college- and non-college households is not targeted in the calibration.

4.9 Government

The government has to balance the budget of the general tax and transfer system as well as the budget of the pension system. In the scope of the general tax and transfer system budget, the government finances an exogenous stream of (non-education related) expenditures and an endogenous stream of education related expenditures (pre-tertiary and tertiary). The revenue side of the general tax and transfer system is comprised by taxes on consumption, capital income and labor income. The consumption tax rate is set to 5% (see Mendoza et al. (1994)) while the capital income tax rate is fixed at 36%, following Trabandt and Uhlig (2011). Additionally, the government can issue debt.

Households that work positive hours in the labor market face the labor income tax schedule that is approximated using a two-parameter tax function as in Heathcote et al. (2017):

$$T(y, n > 0) = y - (1-\tau)y^{1-\xi} \tag{30}$$

where τ is the level parameter, and ξ is the progressivity parameter. The progressivity parameter is exogenously set to 0.18 for all population groups, following Heathcote et al. (2017), while the level parameter is calibrated endogenously to match the government debt to GDP ratio of 100% in the baseline.

The non-participating and unemployed households have no labor income and thus do not pay labor income taxes but receive government transfers ω that are set to 20.2% of average (full-time) earnings (CEX 2001-2007; consumption of bottom 10%). Thus, for non-working singles and couples (i.e. both spouses do not work) the tax/transfer functions are given by:

$$T(q = si, 0) = -\omega, \tag{31}$$

$$T(q = cpl, 0) = -2\omega. \tag{32}$$

If, however, only one spouse is non-working and the other spouse supplies positive hours, the tax function is as follows:

$$T(q = cpl, n(g) > 0, n(g^-) = 0, y) = y - (1-\tau)y^{1-\xi} - \max\{0, 2\omega - (1-\tau)y^{1-\xi}\}. \tag{33}$$

In other words, the government guarantees the minimum income of 2ω also to the couples with only one partner supplying positive hours.

Finally, as for the the pension system, the payroll tax τ^p is set to the current legislative level of 12.4% and the actual progressivity of the pension system is taken into account.

5 Model Validation

There is substantial empirical evidence on the short-run impact of small-scale education reforms; see Bastian and Lochner (2020), García et al. (2020) or the meta-study by Jackson and Mackevicius (2023) for the impact of the social safety net, early childhood interventions, and school funding on child achievement and later-life outcomes, as well as the meta study by Deming and Dynarski (2009) for the effect of college tuition grants on college enrollment and completion.

Before using the model for a counterfactual education policy analysis we view it as crucial to ensure that it has plausible predictions for comparable policy interventions empirically studied and surveyed by this literature. For that, it is important to decide what version and time horizon of our model to confront with these empirical estimates. For data reasons the empirical literature focuses on the short-run effects, and by their small-scale nature the experiments can plausibly be assumed to have no significant impact on the economy-wide interest rate as well as the aggregate and relative wage and the government budget. Therefore we contrast the short-run, partial equilibrium model response to this empirical evidence.[22] Since the range of the empirical estimates is fairly large, our goal is not to argue that our model matches any specific study, but rather to demonstrate that the model-based statistics fall into the empirical range and, especially, does not overstate the positive impact of the education policy reforms discussed in this paper.

5.1 College Tuition Subsidies

The empirical evidence on the short-run, small scale (quasi-)experimental effect of college tuition cost on attendance and completion is quite broad. Deming and Dynarski (2009) summarize the large literature on this topic, with the upshot that an $1,000 increase in college subsidies leads to a 3-6 percentage point increase in college enrollment. Our model implies a response in college enrollment, in partial equilibrium, of 5.1 percentage points.

[22] Partial equilibrium means that wages and interest rates as well as tax rates are held fixed when the policy changes. In contrast, we continue to assume that the marriage market clears, that is, even in partial equilibrium households adjust their beliefs about the characteristics of potential future spouses in response to the policy change.

5.2 Increase in High School Spending

In their meta-study of a large number of empirical (quasi-)experimental studies, Jackson and Mackevicius (2023) report that an increase in public high school funding by $1,000 per pupil for four years leads to an increase in high-school completion by 0.07-3.99 percentage points and an increase in college enrollment by 0.90-5.51 percentage points. We target the midpoint of the estimates for the impact of high-school funding for college enrollment when calibrating the model, but not the response of high school graduation rates to the same intervention, leaving this prediction of the model as an important dimension of model validation. In partial equilibrium of our model, the high school completion rate increases by 0.6 percentage points on impact of a $\$1,000$ increase in public high-school spending, towards the lower bound of the (arguably wide-ranged) empirical estimates, but indicating that in our model public schooling is not "overly" productive relative to the available empirical evidence.

5.3 Discussion

The previous results indicate that our model-implied policy responses to small-scale reforms line up well with the empirical record. Alternatively put, the empirical results reported in the literature are consistent with our structural model, raising the question why we cannot simply extrapolate these empirical estimates of the short-run impact of small reforms to medium- or long-run outcomes at the aggregate, national scale, without any need for structural modelling?

We think the reasons are four-fold. First, policy transitions take time, and the short-run policy effect could be very different from long run impact of the policy since the distribution of the population (from the perspective of our model, with respect to initial assets, human capital and parental education) changes. Section 6.2 below shows that this is indeed the case in our model. Second, a large-scale reform might have important general equilibrium effects on (relative) wages of college and non-college labor as well as rates of returns that are not captured by small-scale quasi-experimental evidence. We show in Section 6.3 that these general equilibrium effects are indeed quantitatively very sizeable. Third, for small-scale reforms the government budgetary consequences can plausibly be neglected, whereas for large-scale, economy-wide reforms the adjustment in taxes, transfers and/or government debt have to be considered explicitly, which requires articulating the intertemporal government budget constraint explicitly (and its adjustment, in the face of an education reform), as we do in Section 6.2.1. Finally, and separately from the first three points that pertain to a *positive* policy analysis, for an assessment of the *normative* consequences of a hypothetical policy reform we need a utility-based structural approach, in our view. We turn to such a model-based positive and normative assessment of the education reforms in the next section.

6 Results for Two Pure Policy Reforms

6.1 The Thought Experiment

We now present the results of our main policy reforms. For each transition thought experiment we assume that the economy is in steady state calibrated to data from 1999 to 2019, and that the policy reform triggering the transition is completely unexpected (the proverbial MIT shock), but that the government is henceforth fully committed to the policy reform. Our benchmark reform is "Free College", a 100% subsidy of college tuition, financed by a permanent increase in labor income tax rate τ. That is, this tax parameter adjusts once and for all to ensure that the intertemporal government budget constraint remains satisfied. In order to guarantee that the period-by-period budget constraint holds, government debt endogenously evolves along the transition from the old steady state towards its new steady state value (as a fraction of GDP). The corresponding "Better Schools" reform increases public (primary and secondary) school spending i^g permanently so that the extra expenditures have the same present discounted value as the "Free College" reform, making both interventions fiscally comparable.

We present our main results in Subsection 6.2, contrasting in turn the aggregate, welfare and distributional (cross-sectional and intergenerational socioeconomic persistence) consequences of the two reforms in general equilibrium. In order to insulate the importance of endogenous factor price movements (that is, changes in (relative) wages as well as the interest rate), in Subsection 6.3 we study the same reforms in a partial equilibrium setting where wages and interest rates remain fixed (so that labor markets and the capital market need not clear, i.e., (17)-(19) need not hold). However, the government intertemporal government budget constraint (43) is required to be satisfied in all our thought experiments.[23]

6.2 General Equilibrium: Transitional Dynamics

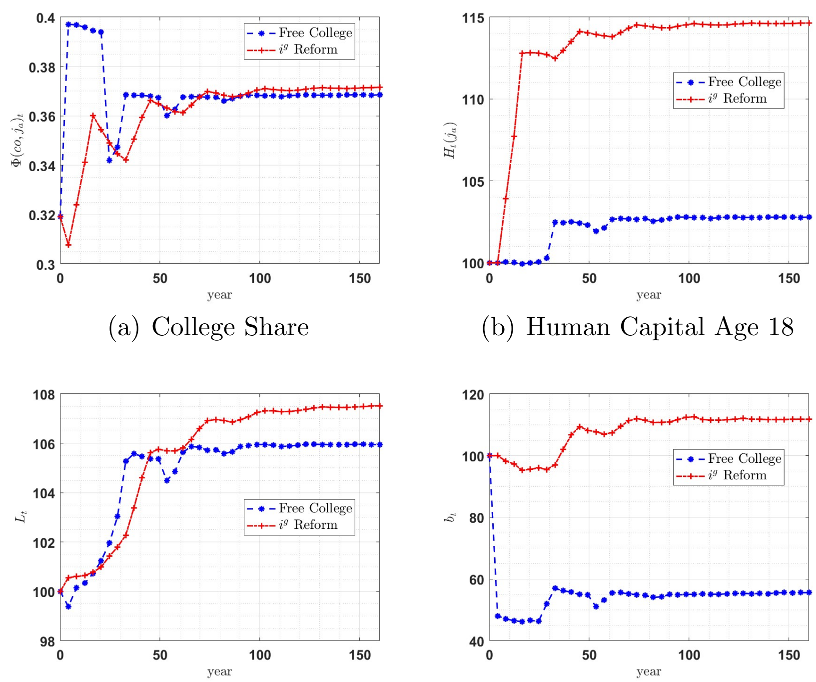
In this section we summarize the transition results from our two main policy exercises; Table 10 in Appendix A provides a summary of comparison of the initial and the final steady states to which the policy transitions converges.

[23] The progressive labor income tax code is specified as a two-parameter family following Benabou (2002) and Heathcote et al. (2017), and for the current thought experiments we fix the progressivity parameter and adjust the tax level parameter once and for all so that the intertemporal government budget constraint is satisfied. The sequence of government debt levels ensures that the flow government budget constraints are satisfied in every period.

6.2.1 Aggregate Effects

In Figure 1 we display the dynamics of the college share, aggregate labor in efficiency units (L_t), average human capital at age 18 and aggregate inter-vivos transfers over time. Panel (a) of Figure 1 demonstrates that both policies are successful in inducing more individuals to attend college, although making college free does so to a larger extent in the short run. However, it also leads to many more college dropouts.[24]. Furthermore, it takes time for the full impact of the reforms to take hold. Only after the third generation born after the reform has made the higher education decision and completed college (i.e., roughly 50 years after the policy change, see Figure 3, panel a, below) does the share of the population with a college degree reach its new, higher steady state level. This broad observation, which also holds true for the other aggregate variables depicted in Figure 1, reinforces the need to model transitions explicitly.

Figure 1: Aggregate Variables: General Equilibrium



(a) College Share — (b) Human Capital Age 18 — (c) Labor — (d) IVT

Notes: Panel (a): Share of a given "birth" cohort that completes college; Panel (b): Average human capital of a given "birth" cohort at age 18 (Initial Steady State = 100); Panel (c): Aggregate labor efficiency units (Initial Steady State = 100); Panel (d) Aggregate Inter-vivos transfers (Initial Steady State = 100).

Panel (b) and (c) of Figure 1 indicate that college expansion is achieved through very different channels in the two reforms. In the "free college" reform, as panel (b) of Figure 1 shows, there

[24]There are also substantial differences in the underlying human capital and, thus, productivity distribution of the pool of new college students induced by each reforms. These differences will be discussed in Section 6.2.2 on the distributional impact of the reforms.

is only a very marginal positive human capital response within the lifespan of the first impacted generation (i.e. before the newly educated children become parents themselves). There are several conflicting forces at play that determine how parental incentives to invest in child human capital are affected by the generosity of the college subsidy. On the one hand, holding fixed the expected benefits from enrolling in college (in terms of graduation probability and earnings), the endogenous optimal human capital attendance threshold is decreasing in the college subsidy rate, creates a disincentive for parents to invest in child pre-college human capital.[25] On the other hand, the benefits from college (in terms of graduation probability and earnings) are increasing in the level of human capital and thus making college financially affordable creates an incentive for altruistic parents to increase their human capital investments in children so that the latter can take bigger advantage of attending college. Quantitatively, the positive and the negative investment incentive effects almost fully offset each other and acquired human capital increases only very marginally within the lifespan of one generation.

In the "better schools" reform there is a much more pronounced increase in child human capital accumulation, and some of the now better-schooled 18 year old teenagers choose to attend college when they used not to. Crucially, as panel (c) demonstrates, even those whose college attendance decisions are not affected by the reform now tend to have more human capital, and consequently are more productive in the labor market. Furthermore, since those attending college now have more human capital, under the "better schools" reform the college completion rate improves as well. Consequently, aggregate labor efficiency units actually rise more strongly under the "better schools" reform than under the "free college" reform.

Finally, panel (d) demonstrates that private parental adjustments also significantly differ: when college is free, private inter-vivos transfers (which are mainly used for financing college tuition) collapse, which in turn reduces overall asset accumulation by parental generations. The strong response in the college share, in aggregate labor as well as aggregate savings also anticipates the finding that accounting for general equilibrium factor price adjustments will have quantitatively very important aggregate, distributional and welfare consequences.[26]

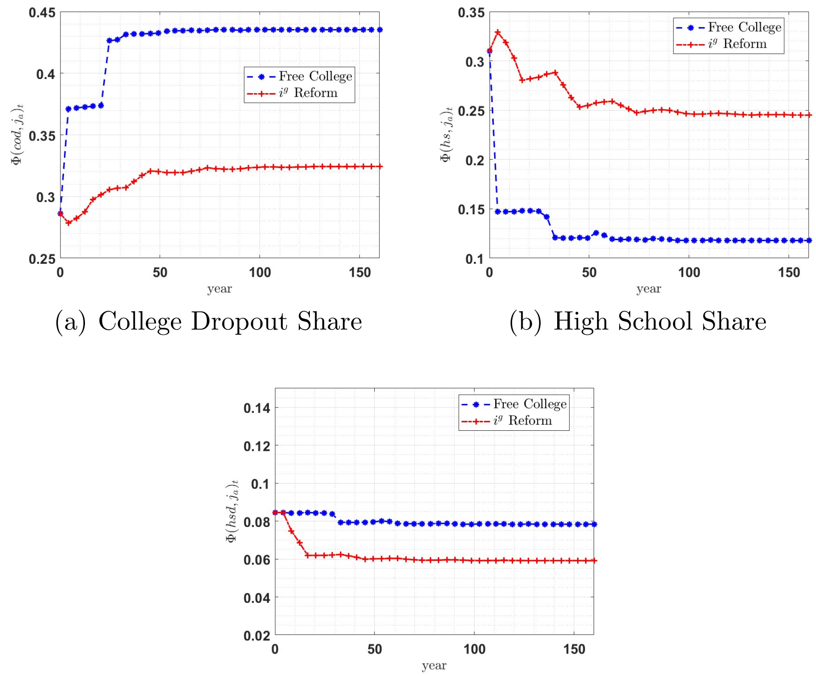
6.2.2 Distributional Consequences

The two policy reforms also have substantially different distributional consequences. This is apparent from Figure 2 which complements panel (a) of Figure 1 and shows the evolution of the

[25] That is, for given other parental state variables, the human capital level starting from which children optimally choose to enroll in college - determined by the psychological cost which is a function of human capital.

[26] The adjustments of parental resource and time investments are shown in Appendix A. The quantitative importance of parental investment adjustments is somewhat limited due to a relatively large relative weight on public schooling in the calibration of the human capital production function.

Figure 2: Higher Education Shares: General Equilibrium



(a) College Dropout Share

(b) High School Share

(c) High School Dropout Share

Notes: Panel (a): Share of a given "birth" cohort that becomes a college dropout; Panel (b): Share of a given "birth" cohort that becomes a high school completer; Panel (c): Share of a given "birth" cohort that becomes a high school dropout.

share of college dropouts in panel (a), those completing high-school in panel (b) and those with some, but not complete college in panel (c). We want to highlight two key observations here.

First, the free college reform does not change the share of children dropping out from high school by much (see panel (c)) even though the incentives of parents to invest time and resources into their children's human capital (to make them potentially successful college students) have increased. For these children, predominantly from families with low parental educational background and often with only a single parent, the problem of college attainability prior to the reform is not (primarily) that it too expensive to attend college, but that their initial and acquired human capital during childhood would make it very strenuous to attend college (the utility cost of attending college is very high, given their human capital) and unlikely to succeed in obtaining a college degree. Anticipating that their children will not go to college, these parents see little reason to change their human capital investment decisions during the child's primary and secondary education years, and thus the share of high-school dropouts only mildly declines under the "free college" reform. In contrast, the "better schools" reform leads to a decline in the share of high-school dropouts in the population by about two percentage points since the larger public investment into child human capital accumulation in school is only partially offset by lower private time and resource investments (see Appendix A.1) The improved human capital distribution at age 16 then results in a smaller share of the population dropping out of high school.

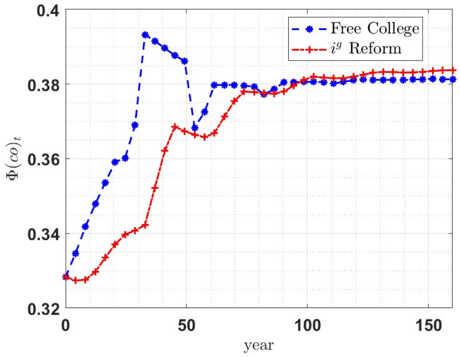
Second, many of the additional students drawn to college under the free college reform do not actually complete college (since their human capital from high school is relatively low and thus the chances of dropping out are high). In the long run (see Table 10 in the appendix), although the "free college" reform shifts 14 percentage points of previous high school graduates to college attendance, only about half of these end up with a degree. In contrast, virtually all of the new college attendees under the "better schools reform" (approximately 5 percentage points) graduate from college, suggesting that this reform uniformly shifts up the tertiary school attainment distribution and benefits all segments of the distribution in terms of labor-market relevant skills.

Figure 3 shows the education *population* shares (as opposed to the education shares of a specific age cohort that was depicted in Figure 1). It displays a recurrent theme of this paper that the education reforms studied in this paper take time to materialize their full effect since the education expansion only directly impacts currently young generations that still have to go through school and/or make their higher education choices. Initially, this is a small share of the population, but over time these cohorts make an increasingly large share of the total labor force and thus the share of college-educated workers gradually increases (and that of individuals with only a high school degree declines). The extent to which this happens differs, of course, across the two reforms and is stronger for the free college thought experiment. In contrast, the better schools reform over time almost halves the share of the population without even a high school degree, although this takes three generations, with no such effect from the free college reform.

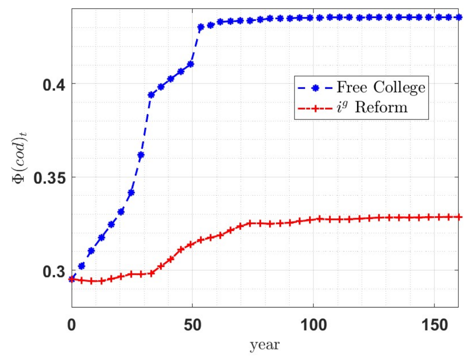
Figure 4 displays the evolution of aggregate capital, output, consumption and government debt. Since capital is only mildly increasing along the transition, the time path of output roughly follows that of aggregate labor input; the same is true for aggregate consumption. For the same reason, the tax base increases gradually with labor along the transition, whereas the education cost in both reforms rises immediately on impact. Therefore, the government accumulates debt along the transition, and since the "better schools" reform delivers a larger output in the long run, the capacity to service debt is more substantial in that reform as well. See panel (d) of Figure 4.

We cast our model in general equilibrium, and therefore interest rates, wages and taxes adjust along the transition path to ensure that the labor markets for college-educated labor, non-college labor and the assets market clears. In Figure 5 we display the time paths of these equilibrium factor prices and taxes. We observe that on account of the increase in labor input (in efficiency

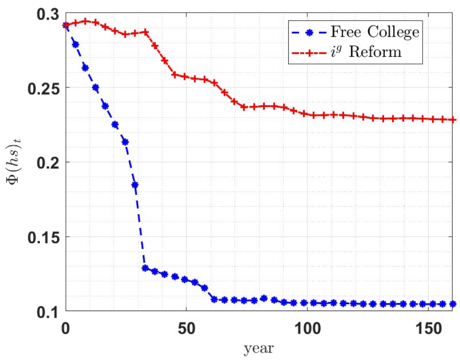
Figure 3: Total Population Education Shares (general equilibrium)



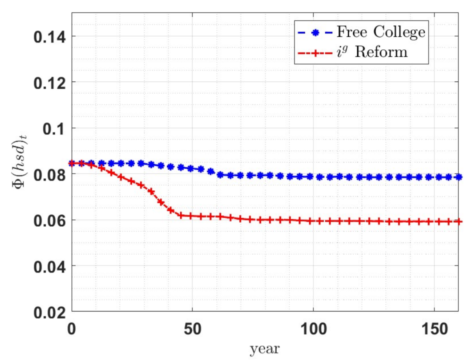
(a) College Share (population)



(b) College Dropout Share (population)



(c) High School Share (population)

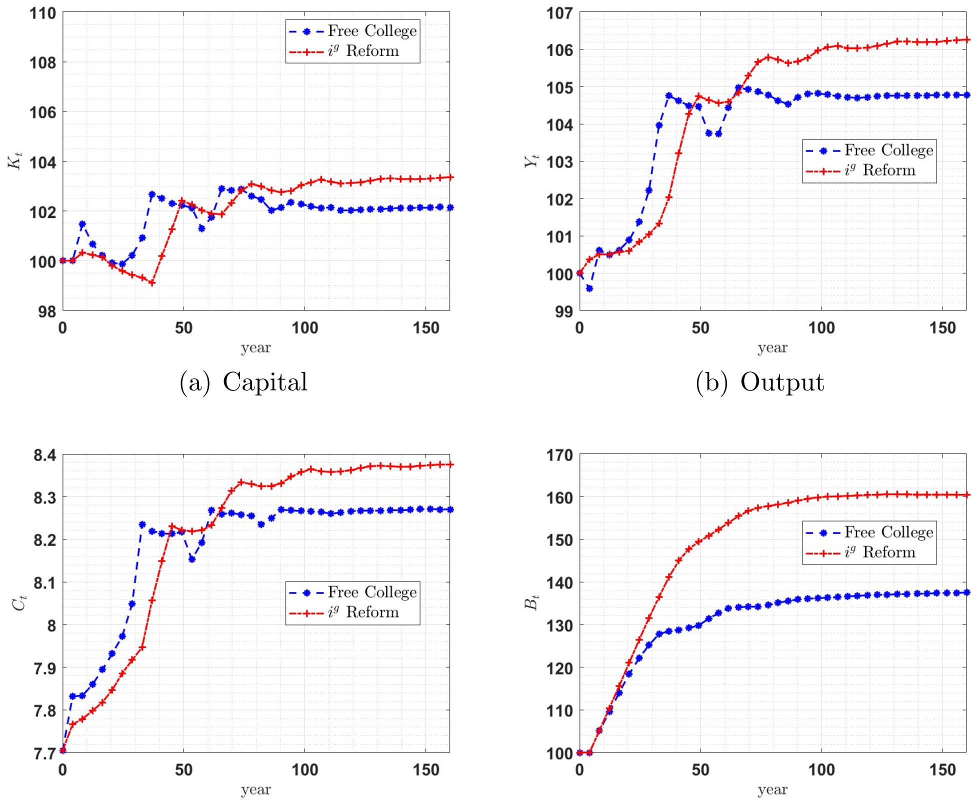


(d) High School Dropout Share (population)

Notes: Panel (a): Population share of college completers; Panel (b): Population share of college dropouts; Panel (c): Population share of high-school completers; Panel (d): Population share of high-school dropouts.

units) induced by both education reforms, the capital-labor ratio falls, the interest rate increases over time (see panel (a)), and wages per efficiency unit (not shown) fall over time. However, since college- and non-college labor are imperfect substitutes and non-college labor becomes scarcer relative to college labor, the college wage premium falls by 10 percentage points under the "free college" reform, and almost 7 percentage points under the "better schools" reform (see panel (b)) but wages of those without a college degree actually increase (see panel (c) of Figure 5). In contrast, those with a college degree see their absolute wages decline substantially (relative to the long-run balanced growth path, of course). Finally, panel (e) shows the (once and for all) adjustment in the labor income tax rate τ required to ensure that the intertemporal government budget constraint holds. It demonstrates that both reforms actually generate more fiscal space in that the reforms are self-financing and the labor income tax rate can actually fall, more so in the "better schools" reform.

Figure 4: Aggregate Consumption, Production and Government Debt



(a) Capital (b) Output

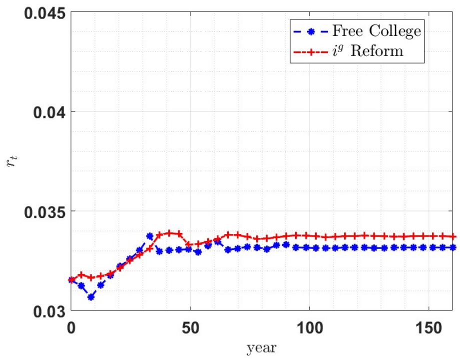
(c) Consumption (d) Debt

6.2.3 Intergenerational Persistence

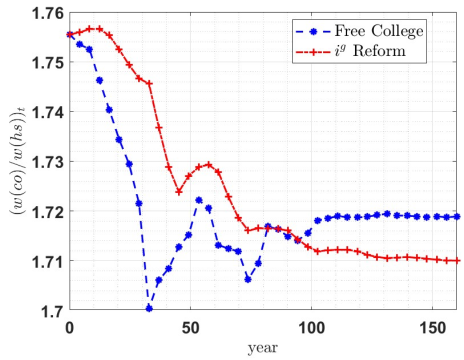
The policy reforms not only affect cross-sectional inequality, but also the intergenerational persistence of earnings and education. Table 7 displays one dimension of intergenerational mobility, showing how the earnings of children from different parental backgrounds (measured by parental education and marital status) change in response to the policy interventions. The first column shows the average lifetime earnings within a specific parental group, i.e., single parents without a high-school degree on average earn $\$21,297$. The second column shows the average earnings of children for each parental group under the baseline policy, and the remaining two columns show the percentage change in these child earnings induced by the two policy reforms.

Table 7 shows that both reforms reduce the earnings gap between socio-economic groups. Interestingly, the reduction is larger in the "free college" reform than in the "better" schools reform because children from the highest socio-economic group overwhelmingly attend college even without it being free, and thus this reform does not induce earnings gains for this group.

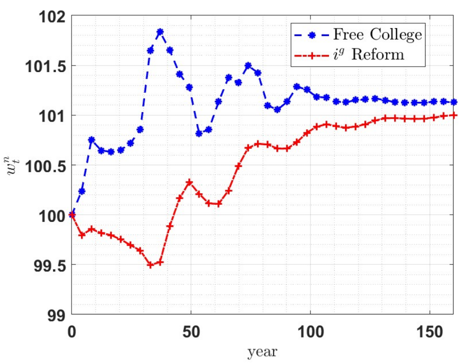
Figure 5: Prices and Taxes in General Equilibrium



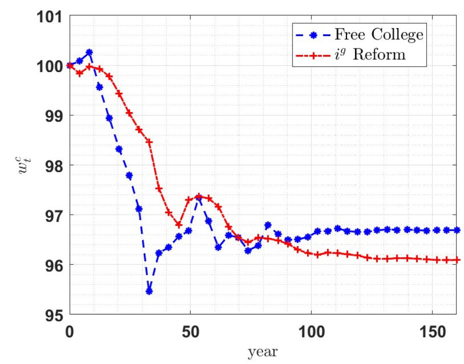
(a) Interest Rate



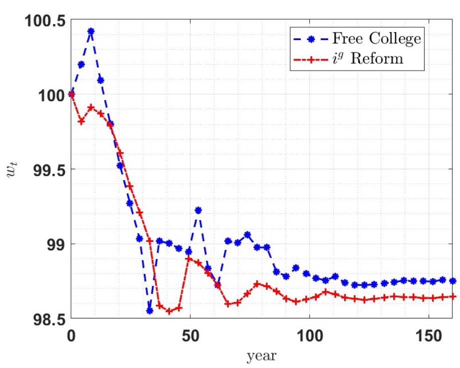
(b) College Wage Premium



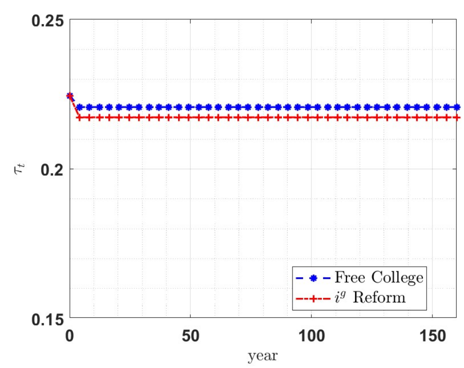
(c) Non-College Wage



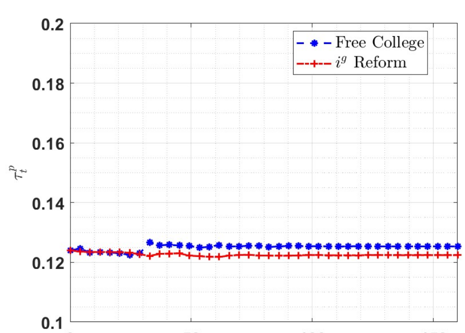
(d) College Wage



(e) Average Wage Level



(f) Labor Income Tax Level Parameter τ



(g) Pension Contribution Rate τ^p

Table 7: Child Earnings % Change, by Parental Background

Parent Back-d	Baseline	Free College, % Δ	Better Schools, % Δ
Single, $s = hsd$ ($21,297)	$45,891	5.52	6.13
Single, $s = hs$ ($37,153)	$56,283	10.45	8.96
Single, $s = cod$ ($44,951)	$59,118	9.31	7.11
Single, $s = co$ ($65,574)	$73,932	-0.40	5.31
Couple, $s = hsd$ ($27,043)	$55,308	9.17	7.35
Couple, $s = hs$ ($44,241)	$57,727	5.63	5.51
Couple, $s = cod$ ($59,484)	$61,332	6.34	4.53
Couple, $s = co$ ($88,968)	$75,242	-0.18	4.43

Notes: Annual gross earnings % change relative to the baseline, averaged over the working life. In parenthesis, own parental (annual, averaged over the working life) earnings are shown.

The "better schools" reform in contrast elevates the accumulation of (earnings-relevant) human capital of all children (including those at the very top). As a result, this reform "raises all boats" and the resulting reduction of child earnings inequality is less pronounced.

Finally, Tables ?? and 8 display the intergenerational transition matrix of educational attainment as well as the changes induced by the policy reforms, separately for single parents (on average the poorest families in the population) and for married parents with dual earners. Table ?? shows the strong intergenerational persistence of education in the benchmark economy: the share of children from parents without a high school degree going to college (and succeeding or dropping out) is only 17% for those with single parents and 22% for those with married parents. In contrast, this number rises to 92% for those with parents that have a college degree (roughly independent of marital status of the parent).

Table 8: Intergenerational Education Transition Matrix: Single Parents

Increased School Funding				
	$s = hsd$	$s = hs$	$s = cod$	$s = co$
$s^p = hsd,\ q = si$	-0.0328	-0.1008	0.0791	0.0545
$s^p = hs,\ q = si$	-0.0283	-0.1009	0.0805	0.0487
$s^p = cod,\ q = si$	-0.0254	-0.0949	0.0967	0.0236
$s^p = co,\ q = si$	-0.0186	-0.0058	-0.0262	0.0506
Free College				
	$s = hsd$	$s = hs$	$s = cod$	$s = co$
$s^p = hsd,\ q = si$	0.0004	-0.3573	0.2653	0.0916
$s^p = hs,\ q = si$	0.0021	-0.3726	0.2868	0.0837
$s^p = cod,\ q = si$	0.0011	-0.3383	0.2826	0.0546
$s^p = co,\ q = si$	0.0006	-0.0058	0.0049	0.0003

Table 8 summarizes one key dimension of the distributional consequences of the educational policy reforms, by showing how the intergenerational education transmission matrices for children with single mothers (that is, the share of children with maternal education $s^p \in \{hsd, hs, cod, co\}$ that end up with own education s) are affected by both policies (in the long run, comparing steady state).[27] Positive percentage point changes relative to the baseline are marked in red, negative changes are marked in blue. The table highlights the very different impact of both reforms on intergenerational persistence of education. A free college reform has virtually no impact on the share of children dropping out of high school (for any parental type). It is successful, however, in drawing a much larger share of those previously only completing high school into college, but close to 3/4 of these additional college goers end up dropping out of college (see the last two columns of the lower panel of Table 8). Since for most teenagers dropping out of college is ex-post inefficient (had they known they would not succeed, they would have opted not to attend college in the first place), the reform is not a very effective intervention of raising the share of the population with a college degree.

In contrast, the better schools reform (upper panel of Table 8) significantly reduces high-school dropout rates (see first column of the table), but it is much less effective shifting previous high-school completers into attempting college. Conditional on going, however, the rise in the dropout rate is much less pronounced than in the free-college reform since teenagers under the better schools reform are much better prepared for college (in the sense of having higher human capital which translates into lower dropout probabilities). The results from both reform displayed here also suggest that a mixed reform that uses some of the budget to improve schools to make children more college ready and make it cheaper to attend (albeit not necessarily free) could attain higher attendance without massively increasing dropout, and thus achieve the best of both worlds. This is in fact what we will demonstrate in Section 7 on the optimal (within a restricted policy set) policy reform.

6.2.4 The Welfare Consequences of the Reforms

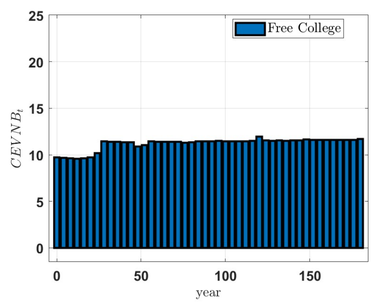
Figure 6 displays the welfare consequences of both policy reform transitions, measured as consumption equivalent variation of economically newborn individuals (i.e., based on expected lifetime utility at age 18), and plotted as a function of the period of the transition at which these individuals enter the economy (i.e., $t = 0$ means individuals becoming economically active in the first period of the transition). Specifically, we ask what uniform (across individual types -initial assets, human capital and parental education-, across time and states of the world) increase of consumption households born into the old steady state would require to be indifferent between

[27] The corresponding results for married parents are contained in Appendix A.3.

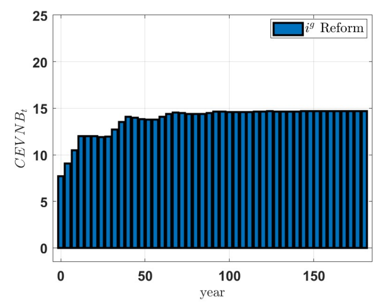
the status quo steady state and to being born into the transition induced by the policy reform. The left panels are for the free college reform and the right panels are for the expansion of public school funding.

In order to distinguish the welfare gains originating from newborns living a better life (i.e., having a larger value function for a given initial state (a, h, s_p) of a newborn) from an improved (or worsened) distribution over these initial state variables (Φ_t vs. Φ_0), in the lower panels we display the welfare consequences that would emerge if the (endogenous) distribution over initial characteristics were to remain unchanged at Φ_0. Thus, the lower panel captures purely the welfare gains from higher lifetime utilities of the different types of economically newborn individuals, and the difference between the upper and the lower panels therefore reflects the welfare consequences of the endogenous and policy-induced shift in the distribution of the initial characteristics (human capital, financial wealth and parental education (a, h, s_p)) of the 18-year olds.[28]

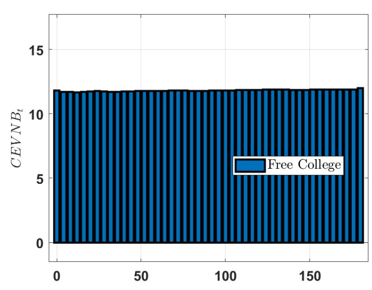
Figure 6: Welfare Gains of Newborns



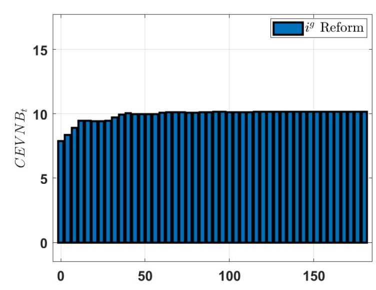
(a) CEV Newborns: Free College



(b) CEV Newborns: Pre-College Funding



(c) CEV NB: Free College, Φ_0



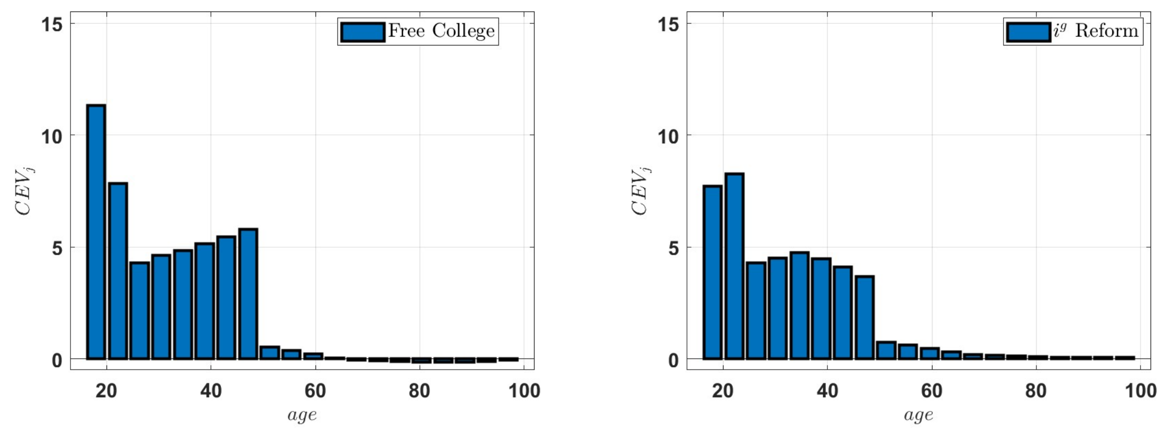
(d) CEV NB: Pre-College Funding, Φ_0

We highlight three qualitative points. First, both reforms entail substantial welfare gains for current newborns and future generations. In contrast to aggregate allocations, the welfare gains are fairly smooth across generations along the transition. The availability of government debt allows the government to smooth the short-run costs and use the long-run higher tax revenues

[28]It is understood that the lifetime utilities of newborns are also affected by changes in the marriage market distribution.

from the education reform to make transition "painless" for newborns (and the majority of the currently alive). Second, as a comparison between the upper and the lower left panel reveals, the direct benefits of "free college" for 18 year-olds are partially offset by the fact that they enter adult life with fewer assets as their parents respond to the policy by adjusting inter-vivos transfers. That is, the actual welfare gains for the youth are smaller compared to a scenario where the policy is evaluated under a fixed initial distribution of wealth (as well as human capital), especially early in the transition.[29] Third, and in sharp contrast, the key part of the welfare gains with better school funding comes from an improved human capital and parental education distribution and these gains increase over time, as can be seen comparing the top and the bottom right panels. This reinforces the need for studying (debt financed) policy *transitions* when considering fundamental education reforms.

Figure 7: Welfare Gains of Currently Alive Population



(a) CEV Currently Alive, Total Population: "Free College"

(b) CEV Currently Alive, Total Population: "Better Schools"

Of course, welfare gains for academically newborn agents might partially come at the cost of welfare losses for existing (at the time of the policy reform) generations that potentially have to pay the higher taxes but do not benefit directly from the reforms since their human capital accumulation and tertiary education decisions lie in the past. However, since these generations are altruistically motivated toward their children, these generations might benefit indirectly through higher expected lifetime utility of their offspring. They are also affected by GE price adjustments.

In Figure 7 we summarize the welfare consequences (again measured in terms of consumption equivalent variation) of these generations (by their age, and again averaged over their relevant state variables). Again the left panel is for the free college reform and right panel for the better school reform. We observe that younger generations with children still in the household also gain,

[29] Along the transition, the fact that young adults have better educated parents in part offsets this effect.

mostly on account of the higher lifetime utility of their children. Older generations (those age 48 and older whose college education is completed and whose children have left the households) have smaller welfare gains, and if they are retired, might actually suffer welfare losses. This is due to general equilibrium effects: when wages fall, so do benefits from the PAYGO social security system, which offset (and for older households, dominate) the mild increase in asset returns and (if still in working age) the reduction in the labor income tax rate. Note, however, that these welfare losses are relatively mild. Thus, although neither reform constitutes a Pareto improvement (since the current old lose), it is conceivable that a reform that phases in the tax increases slowly might be sufficient to avoid the welfare losses for generations older than 48 at the time of the reform that Figure 7 documents.[30]

6.3 Decomposition of General Equilibrium Effects

To isolate the importance of changes in endogenous interest rates and (relative) wages we also conduct a sequence of partial equilibrium exercises in which we hold these endogenous prices as well as the taxes required to balance the intertemporal government budget constant. As a summary, we show that qualitatively, the aggregate and to a large degree the distributional conclusions discussed above also emerge in the absence of equilibrium price adjustments. However, endogenous interest and (relative) wage adjustments in general equilibrium make the welfare gains for newborn generations smaller (relative to partial equilibrium) and reduce the difference between the two reforms.

The most important general equilibrium effects stem from the fact that inflow of more college-educated workers into the labor market (induced by the education reforms) and their higher human capital lowers both the capital-labor ratio and the college wage premium, in turn muting the increase in the college share in general equilibrium relative to partial equilibrium. The decline in the capital-labor ratio puts downward pressure on all wages (which hurts workers) but raises the interest rate. The *relative* wage effect, which provides welcome (from the perspective of ex-ante utility) redistribution across education types, is stronger in the school expenditure expansion reform since college enrollment decisions are more sensitive to the college wage premium in that thought experiment.

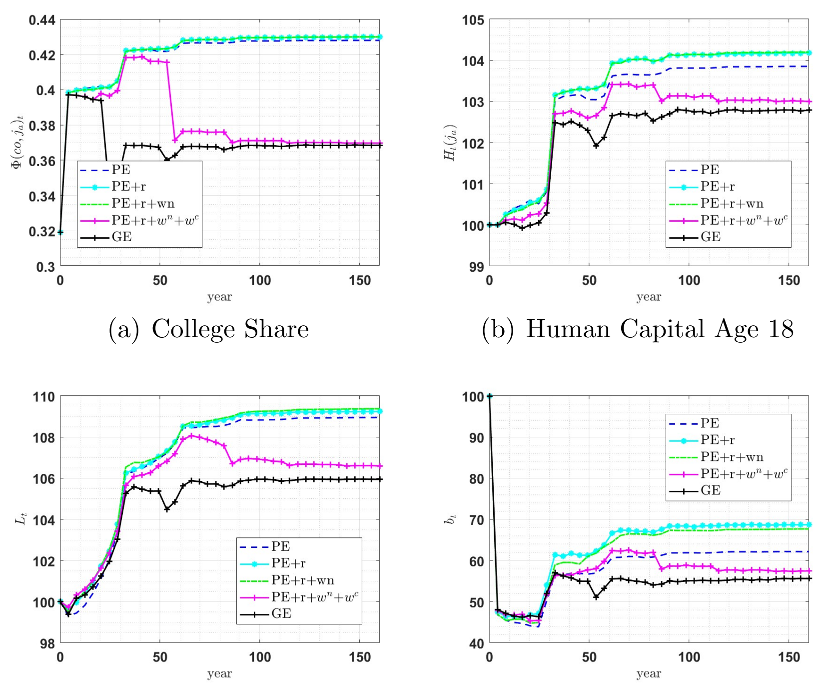
In addition, an increase in the interest rate induced by the reduction of the capital-labor ratio (in turn due to the increase in effective labor as well the reduced savings incentives for privately funded education expenditures and inter-vivos transfers and the shift from capital to government

[30] Although not necessary here, in Section 7 we will introduce specific social welfare functions to aggregate the welfare gains and losses documented here, and to determine (according to those social welfare functions) the optimal mix of both policies. Not surprisingly, the aggregated welfare gains lie in between those experienced by the newborns and existing old generations.

debt) in general equilibrium mutes the crowding-out effect of the free college reform on inter-vivos transfers and results in a larger crowding-in under the school expenditure reform. Overall, as a result of these general price movements and required tax adjustments, the welfare gains are smaller in general relative to partial equilibrium, and the *difference* between the two reforms is smaller in general equilibrium relative to partial equilibrium as well.

In Figure 8 we display the college share, human capital at age 18, aggregate labor in efficiency units as well as inter vivos transfers over time for the free college reform for five scenarios. The first is our general equilibrium benchmark experiment. The second scenario labeled partial equilibrium ("PE") holds all prices (wages and interest rates) constant, but adjusts the level tax parameter τ once and for all so that the intertemporal government budget constraint continues to hold in partial equilibrium. The remaining three scenarios departs from the PE scenario, but sequentially feed in the GE interest path ("PE+r"), then also the wage path for non-college labor ("PE+$r + w^n$") and then also the wage for college labor ("PE+$r + w^n + w^c$"). The only difference between this last scenario and the "GE" scenario is the latter has a higher labor income tax rate, relative to the tax level needed in partial equilibrium. These thought experiments seek to isolate, separately, the importance of changes in wages and changes in the real interest rate induced by the free college reform. Figure 9 does the same for the "Better Schools" reform.

Figure 8: Free College Reform, Aggregate Variables: GE Decomposition



(a) College Share

(b) Human Capital Age 18

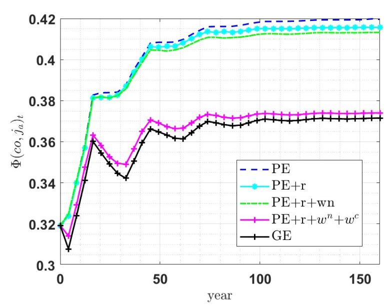
(c) Labor

(d) IVT

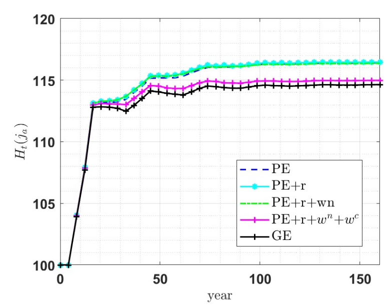
Very broadly, and with some nuance for a subset of the variables, the key general equilibrium effect comes from the endogenous adjustment of college-educated labor. This is apparent from the upper left panel of the figure. The partial equilibrium response of the college share is large and increasing over time, and adjustments in the interest rates do not change that finding (as the three lines are virtually on top of each other). However, when the college wage adjusts downward (as it does in GE), after an initial massive increase this share falls and settles down at a somewhat higher level relative to the pre-reform scenario. The (relatively small) change in the tax rate does not much affect this conclusion that it is the decline in the college wage (and thus the college wage premium) in general equilibrium that enacts the largest impact on the college share. This is also the driving force behind the smaller welfare gains of both reforms in general- relative to partial equilibrium. As Figures 12 and 13 as well as Table 10 in the Appendix show that, as a result of these general price movements, the welfare gains are 2.6-4.6 percentage points smaller, and the *difference* between the two reforms is also less by 2 percentage points.

The endogenous interest rate increase (see again panel (a) of Figure 5) has only a minor effect on the college share, as the difference between the PE and the "PE+r" line in panel (a) is negligible. In contrast, the rise in the interest rate is more important for the intergenerational transmission of wealth, as panels (d) of Figures 8 and 9 display. Comparing the blue "PE" line with the cyan "PE+r" line shows that the increase in the interest rate in general equilibrium mitigates (but not fully offsets) the decline in inter-vivos transfers that the education reforms, especially the free college reform would otherwise have induced.

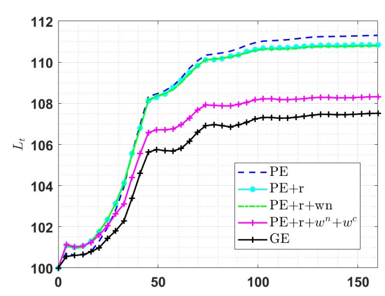
Figure 9: Better Schools Reform, Aggregate Variables: GE Decomposition



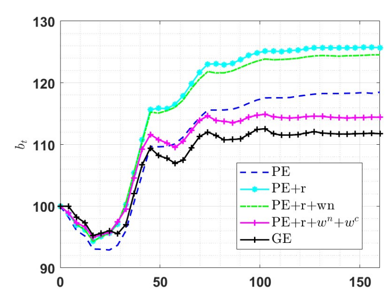
(a) College Share



(b) Human Capital Age 18



(c) Labor



(d) IVT

7 Optimal Policy

Thus far, we have considered two "pure" policy reforms in isolation, and we have seen that both generated significant welfare gains. However, we have also documented that especially for children from poor families college remains largely out of reach since under a free college reform the low human capital acquired in primary and secondary school translates into college failure rates that are so high to make college unattractive even if free. The pure "better schools" reform, in contrast, raises human capital of the entire population, including that of poor children, but without some college tuition subsidies going to college remains too expensive for the poorest children.

This raises the question whether a combination of both reforms raises human capital of these children sufficiently to make college success feasible while sufficiently reducing the cost to make it affordable. Therefore, in this section we seek to characterize the optimal combination of a "better schools" reform and a college tuition subsidy policy, holding the total cost of the reform constant at the level in the previous section. Specifically, let Ω be the share of the total additional government education expenditures be allocated to tuition subsidies and $1-\Omega$ be the share devoted to better schools. The reforms in the previous section correspond to $\Omega = 1$ ("free college") and $\Omega = 0$ ("better schools). We now seek to find the $\Omega^* \in [0,1]$ that maximizes social welfare $\mathcal{W}(\otimes)$.

7.1 Measurement of Social Welfare

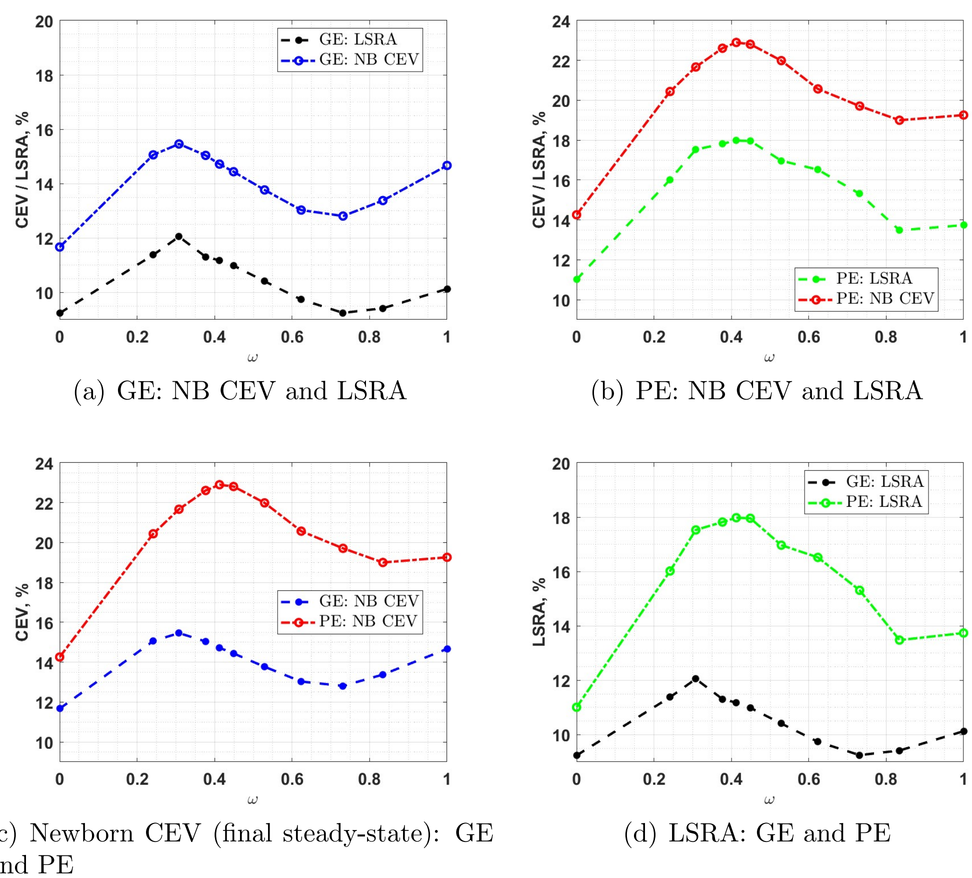
In our life cycle economy with current and future generations and heterogeneity within generations the choice of the social welfare function $\mathcal{W}(\otimes)$ is not obvious. We will consider two alternatives, chosen again to highlight the potential distinction and conflict between the policy impact in the short- and in the long run. Our long-run welfare measure is the expected lifetime utility of economic newborns in the steady state, where as before expectations are taken under the veil of ignorance, that is, before the initial state (a, h, s_p) is realized. This was the welfare measure the discussion in the previous section has focused on.

The second alternative that captures the welfare consequences both for generations living through the transition and those born into the new steady state, is based on the lump-sum redistribution authority concept initially introduced by Auerbach and Kotlikoff (1987) and further refined by, e.g., Boar and Midrigan (2022). Specifically, for each individual currently alive we compute the (possibly negative) wealth transfer required to make the individual indifferent between the status quo and a specific reform. We do the same for all newborn generations along the transition path and in the new steady state. We then aggregate these transfers using the populations shares in the initial steady state (for individuals already alive) and the relative size of newborn agents (relative to the population in the initial steady state), and discount transfers along the transition using the initial equilibrium interest rate (so that the discounting is the same for all policy reforms considered). Finally, for comparison with other welfare measure we translate the aggregate wealth transfer into a flow consumption measure. For details how the LSRA welfare measure is constructed, see Appendix B.8.

7.2 The Optimal Policy Mix

In Figure 10 we plot the welfare gains, measured as CEV of newborns in the steady state and (flow-consumption based LSRA) as a function of the policy weight Ω, where again $\Omega = 0$ is the free college reform and $\Omega = 1$ corresponds to the "better schools" reforms discussed in the previous section. We do so for the general equilibrium benchmark model and, in order to interpret results, for the partial equilibrium version of the model.

Figure 10: Joint Reforms in GE and PE: Welfare



(a) GE: NB CEV and LSRA

(b) PE: NB CEV and LSRA

(c) Newborn CEV (final steady-state): GE and PE

(d) LSRA: GE and PE

Notes: Ω (on the x-axis) denotes the relative weight on the i^g reform.

Although the precise optimum Ω^* evidently depends on the adopted welfare criterion, three robust findings emerge. First, and consistent with the previous section, for all weights Ω the welfare gains of the policy reforms are quite sizable (in the order of above 10% of lifetime consumption) and larger in partial equilibrium than in general equilibrium (see panels (c) and (d) which compare the welfare gains in partial and general equilibrium for the CEV and the LSRA welfare measure, respectively). Second, the CEV welfare measure that focuses on steady states shows larger gains than the ones that includes transitional generations (the LSRA measure), again confirming that the full welfare benefits of the reforms take time to materialize.

Finally, and most importantly for the purpose of this section, the highest welfare is attained in the interior of the policy space and spends ca. $1/3$ of its budget on better schools and the remainder on subsidizing the cost of college.

Table 9 displays the change in intergenerational education persistence induced by the optimal policy mix; it is the counterpart of Table 8 and shows that the mixed reform is almost as successful as the pure better schools reform in curbing dropping out of high-school, while almost as successful as the free college reform in strengthening college completion, albeit at the cost of some additional college drop-outs (which is significantly less pronounced than in the pure free college reform). In this sense, and within the same fiscal budget, the mixed reform achieves the "best of both worlds", with resulting welfare gains that surpass both pure reforms as shown above.

Table 9: Intergenerational Education Transition Matrix: Optimal Mix

Single Parents				
	$s = hsd$	$s = hs$	$s = cod$	$s = co$
$s^p = hsd$, $q = si$	-0.0220	-0.2035	0.1368	0.0887
$s^p = hs$, $q = si$	-0.0191	-0.1811	0.1334	0.0668
$s^p = cod$, $q = si$	-0.0174	-0.1711	0.1315	0.0570
$s^p = co$, $q = si$	-0.0124	-0.0102	-0.0119	0.0345

8 Conclusion

In this paper we studied the optimal combination of college tuition subsidies and school financing, for a given pre-specified budget for these reforms. We evaluated the aggregate, distributional and welfare consequences of these reforms targeted at different stages of childhood and adolescence. We find that although individual reforms generate very significant welfare gains, a combination of both is most effective, in a welfare sense, of both curbing high-school dropout rates and encouraging college attendance without an overly large increase in college dropout rates.

Our analysis held both the overall size of the education reforms constant, and documented that it is more than budget-neutral in an intertemporal sense, but required temporary deficits and thus an increase in government debt along the transition. An natural extension would be to analyze the quantitative importance of access to additional government debt, by either considering a world in which the reform has to be financed period by period by tax changes (which would be tax increases in the short run). Furthermore, an analysis of the optimal *size* of the reform(s), both in the presence and the absence of a period-by-period budget balance assumption, would be informative about the importance of the "fiscal space" for the success of education reform.

More broadly, we have taken the remainder of the fiscal constitution, that is, the tax-transfer system as given and invariant to the education policy reform. However, especially changes in the welfare system making transfers to the poor (which they might be used for additional education investments by the impacted families) could provide an alternative mobility enhancing policy. A quantitative analysis of a more comprehensive reform of the entire tax-transfer-education financing system, with specific focus on the performance of children of single mothers that constitute more than 20% of the current US child population[31] and are subject to particularly low upward mobility levels, would be especially relevant in this context. We defer this to ongoing and future work.

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[31] See e.g. the 2019 Pew Research Center study on "Religion and Living Arrangements Around the World"

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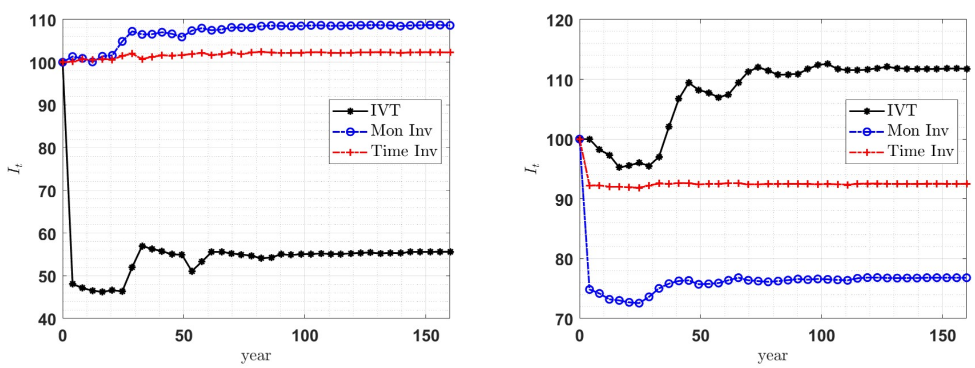
A Additional Quantitative Results

A.1 Parental Investment Responses: "Free College" and "Better Schools"

The optimality condition linking parental resource and time input choices is derived in Appendix B. The plots below show the average per child parental inputs in terms of money, time as well as inter-vivos transfers for the two main reform scenarios. These are aggregate parental input adjustments in the education reform induced transitions. In both reform scenarios, resource investment in children responds stronger in the aggregate than the time investment. From the optimality condition linking monetary and time inputs, $\frac{i_m}{i_t^{1+\psi}}$ is a decreasing function of the marginal utility of consumption. This relation at the individual level, aggregated up, moves in the same direction as the aggregate consumption. Given the optimal consumption responses, the ratio $\frac{i_m}{i_t^{1+\psi}}$ moves up in the aggregate, but the ratio $\frac{i_m}{i_t}$ moves slightly down, i.e. the positive consumption response would need to be even stronger to make the $\frac{i_m}{i_t}$ ratio move upward.

Figure 11: Parental Investments



(a) Interg. Investments: Free College

(b) Interg. Investments: i^g Reform

A.2 Comparison of Steady States: Summary Tables

Table 10: Aggregates, Prices, Taxes and Welfare: Univariate Reforms

Variable	Initial SS	Δ GE FC	Δ PE FC	Δ GE i^g	Δ PE i^g
$\Phi(j_a, s = co)$	31.93%	4.92	10.87	5.23	10.04
$\Phi(j_a, s = cod)$	28.61%	14.92	10.06	3.85	3.64
$\Phi(j_a, s = hs)$	31.01%	-19.23	-20.15	-6.54	-10.88
$\Phi(j_a, s = hsd)$	8.45%	-0.61	-0.79	-2.54	-2.79
HK	1.00	2.78	3.86	14.64	16.41
L	8.85	5.94	8.90	7.57	11.34
Hours	0.28	1.09	1.83	2.36	3.77
C	7.70	7.35	9.67	8.74	11.59
K	12.55	2.00	-9.16	3.42	-9.62
B	3.38	36.54	64.30	58.54	89.23
Revenues	2.54	9.77	11.22	12.54	13.08
Y	13.22	4.72	3.46	6.31	4.93
r	0.13	5.57	0.00	7.32	0.00
w	0.74	-1.28	0.00	-1.34	0.00
$\frac{w^c}{w^n}$	1.07	-4.38	0.00	-4.88	0.00
w^n	0.97	1.11	0.00	1.01	0.00
w^c	1.04	-3.32	0.00	-3.92	0.00
τ	0.22	-1.72	-5.40	-3.24	-8.63
τ^p	0.12	1.09	-0.45	-1.27	-3.43
$\frac{T(AE_0)}{AE_0}$	0.19	-0.40	-1.26	-0.76	-2.02
$\frac{\text{Lab. Inc. Tax Rev}}{Y}$	11.34%	0.28	1.16	0.23	0.98
CEV NB	0.00%	11.17	14.26	14.68	19.26
CEV alive	0.00%	2.45	2.72	2.23	2.89
LSRA	0.00%	9.24	11.01	10.13	13.74

Notes: GE refers to the full general equilibrium version of the model where wages and interest rate are endogenous as well as the government budget constraints are balanced. The PE version refers to a lifecycle model with balanced government budgets, with aggregate physical capital being computed as: $K_t = A_t - B_t$, where A_t are total household assets, and B_t is debt stock (which is endogenous and results from rebalancing the intertemporal government budget constraint).

Table 11: Aggregates, Prices, Taxes and Welfare: Free College

Variable	Initial SS	Δ PE	Δ PE+r	Δ PE+$r + w(s < co)$	Δ PE+$r + w(s < co) + w(co)$	Δ GE
$\Phi(j_a, s = co)$	31.93%	10.87	11.07	11.07	5.04	4.92
$\Phi(j_a, s = cod)$	28.61%	10.06	10.10	10.10	14.98	14.92
$\Phi(j_a, s = hs)$	31.01%	-20.15	-20.31	-20.30	-19.36	-19.23
$\Phi(j_a, s = hsd)$	8.45%	-0.79	-0.86	-0.86	-0.65	-0.61
HK	1.00	3.86	4.18	4.20	3.00	2.78
L	8.85	8.90	9.19	9.32	6.57	5.94
Hours	0.28	1.83	2.16	2.41	2.09	1.09
C	7.70	9.67	11.80	12.28	8.81	7.35
K	12.55	-9.16	0.67	1.57	-2.96	2.00
B	3.38	64.30	64.29	64.29	64.29	36.54
Revenues	2.54	11.22	15.60	16.61	8.68	9.77
Y	13.22	3.46	7.62	8.39	3.52	4.72
r	0.13	0.00	5.57	5.57	5.57	5.57
w	0.74	0.00	0.00	0.00	-1.28	-1.28
$\frac{w^c}{w^n}$	1.07	0.00	0.00	0.00	-4.38	-4.38
w^n	0.97	0.00	0.00	1.11	1.11	1.11
w^c	1.04	0.00	0.00	0.00	-3.32	-3.32
τ	0.22	-5.40	-3.24	-3.24	-3.24	-1.72
τ^p	0.12	-0.45	-1.27	-1.27	-1.27	1.09
$\frac{T(AE_0)}{AE_0}$	0.19	-1.26	-0.76	-0.76	-0.76	-0.40
$\frac{\text{Lab. Inc. Tax Rev}}{Y}$	11.34%	1.16	0.71	tbc	0.15	0.28
CEV NB	0.00%	14.26	16.95	17.79	13.41	11.17
CEV alive	0.00%	2.72	3.29	3.51	2.96	2.45
LSRA	0.00%	11.01	12.88	tbc	10.61	9.24

Notes: GE refers to the full general equilibrium version of the model where wages and interest rate are endogenous as well as the government budget constraints are balanced. The PE version refers to a lifecycle model with balanced government budgets, with aggregate physical capital being computed as: $K_t = A_t - B_t$, where A_t are total household assets, and B_t is debt stock (which is endogenous and results from rebalancing the intertemporal government budget constraint).
PE+r refers to the PE version where the interest rate path from the GE version is exogenously imposed - without any adjustments of other outerloop variables. Therefore, the government budget constraints do not have to hold, and therefore the debt values (as well as the implied physical capital $K_t = A_t - B_t$ and thus also output are not shown, and marked as n/a.
PE+r+$w(s < co)$+$w(co)$ refers to the PE version where the interest rate and wages paths from the GE version are exogenously imposed - without any adjustments of other outerloop variables.

Table 12: Aggregates, Prices, Taxes and Welfare: Better Schools

Variable	Initial SS	Δ PE	Δ PE+r	Δ PE+$r+w(s<co)$	Δ PE+$r+w(s<co)+w(co)$	Δ GE
$\Phi(j_a, s=co)$	31.93%	10.04	9.64	9.39	5.48	5.23
$\Phi(j_a, s=cod)$	28.61%	3.64	2.98	2.83	4.18	3.85
$\Phi(j_a, s=hs)$	31.01%	-10.88	-9.84	-9.45	-7.07	-6.54
$\Phi(j_a, s=hsd)$	8.45%	-2.79	-2.79	-2.78	-2.59	-2.54
HK	1.00	16.41	16.45	16.37	14.98	14.64
L	8.85	11.34	10.81	10.77	8.33	7.57
Hours	0.28	3.77	3.44	3.69	3.36	2.36
C	7.70	11.59	13.86	14.19	10.87	8.74
K	12.55	-9.62	2.92	3.68	-1.22	3.42
B	3.38	89.23	89.34	89.34	89.34	58.54
Revenues	2.54	13.08	17.69	18.35	10.62	12.54
Y	13.22	4.93	9.65	10.20	5.24	6.31
r	0.13	0.00	7.32	7.32	7.32	7.32
w	0.74	0.00	0.00	0.00	-1.34	-1.34
$\frac{w^c}{w^n}$	1.07	0.00	0.00	0.00	-4.88	-4.88
w^n	0.97	0.00	0.00	1.01	1.01	1.01
w^c	1.04	0.00	0.00	0.00	-3.92	-3.92
τ	0.22	-8.63	-8.63	-8.63	-8.63	-3.24
τ^p	0.12	-3.43	-3.36	-3.90	-1.53	-1.27
$\frac{T(AE_0)}{AE_0}$	0.19	-2.02	-2.02	-2.02	-2.02	-0.76
$\frac{\text{Lab. Inc. Tax Rev}}{Y}$	11.34%	0.98	0.34	tbc	-0.21	0.23
CEV NB	0.00%	19.26	22.42	23.05	18.27	14.68
CEV alive	0.00%	2.89	3.61	3.60	3.16	2.23
LSRA	0.00%	13.74	15.89	tbc	13.20	10.13

Notes: GE refers to the full general equilibrium version of the model where wages and interest rate are endogenous as well as the government budget constraints are balanced. The PE version refers to a life cycle model with balanced government budgets, with aggregate physical capital being computed as: $K_t = A_t - B_t$, where A_t are total household assets, and B_t is debt stock (which is endogenous and results from rebalancing the intertemporal government budget constraint).
PE+r refers to the PE version where the interest rate path from the GE version is exogenously imposed - without any adjustments of other outer loop variables. Therefore, the government budget constraints do not have to hold, and therefore the debt values (as well as the implied physical capital $K_t = A_t - B_t$ and thus also output are not shown, and marked as n/a.
PE+r+$w(s < co)$+$w(co)$ refers to the PE version where the interest rate and wages paths from the GE version are exogenously imposed - without any adjustments of other outer loop variables.

Table 13: Aggregates, Prices, Taxes and Welfare: Optimal Mix

Variable	Initial SS	Δ PE	Δ PE+r	Δ PE+$r+w(s<co)$	Δ PE+$r+w(s<co)+w(co)$	Δ GE
$\Phi(j_a, s=co)$	31.93	14.11	14.07	13.88	7.88	7.25
$\Phi(j_a, s=cod)$	28.61	10.53	10.44	10.28	13.20	13.20
$\Phi(j_a, s=hs)$	31.01	-22.79	-22.63	-22.30	-19.46	-18.95
$\Phi(j_a, s=hsd)$	8.45	-1.85	-1.87	-1.86	-1.62	-1.51
HK	1.00	9.84	9.95	9.92	8.35	7.69
L	8.85	12.99	12.91	12.87	9.67	8.55
Hours	0.28	3.57	3.48	3.65	3.63	2.29
C	7.70	14.50	16.96	17.12	13.11	10.50
K	12.55	-9.00	3.23	3.71	-2.00	4.03
B	3.38	91.72	91.76	91.76	91.76	57.15
Revenues	2.54	13.15	18.00	18.32	9.59	12.12
Y	13.22	6.42	11.27	11.57	5.91	7.17
r	0.13	0.00	7.72	7.72	7.72	7.72
w	0.74	0.00	0.00	0.00	-1.45	-1.45
$\frac{w^c}{w^n}$	1.07	0.00	0.00	0.00	-4.86	-4.86
w^n	0.97	0.00	0.00	0.93	0.93	0.93
w^c	1.04	0.00	0.00	0.00	-3.97	-3.97
τ	0.22	-12.48	-12.48	-12.48	-12.48	-5.73
τ^p	0.12	-1.94	-1.91	-2.23	-0.22	0.28
$\frac{T(AE_0)}{AE_0}$	0.19	-2.92	-2.92	-2.92	-2.92	-1.34
$\frac{\text{Lab. Inc. Tax Rev}}{Y}$	11.34	0.84	0.16	tbc	-0.47	0.21
CEV NB	0.00	21.67	24.84	25.18	20.27	15.48
CEV alive	0.00	4.05	4.76	4.89	4.28	3.10
LSRA	0.00	18.01	21.42	tbc	15.69	11.68

Notes: GE refers to the full general equilibrium version of the model where wages and interest rate are endogenous as well as the government budget constraints are balanced. The PE version refers to a lifecycle model with balanced government budgets, with aggregate physical capital being computed as: $K_t = A_t - B_t$, where A_t are total household assets, and B_t is debt stock (which is endogenous and results from rebalancing the intertemporal government budget constraint).
PE+r refers to the PE version where the interest rate path from the GE version is exogenously imposed - without any adjustments of other outerloop variables. Therefore, the government budget constraints do not have to hold, and therefore the debt values (as well as the implied physical capital $K_t = A_t - B_t$ and thus also output are not shown, and marked as n/a.
PE+r+$w(s < co)$+$w(co)$ refers to the PE version where the interest rate and wages paths from the GE version are exogenously imposed - without any adjustments of other outerloop variables.

A.3 Intergenerational Persistence of Education: Married Parents

Table 14 shows the change in the intergenerational education state transition matrix for children with married parents.[32] It shows the same qualitative pattern as for children with single parents that we report in the main text.

Table 14: Intergenerational Education Transition Matrix: Married Parents

Increased School Funding				
	$s = hsd$	$s = hs$	$s = cod$	$s = co$
$s^p = hsd$, $q = cpl$	-0.0295	-0.0505	0.0516	0.0284
$s^p = hs$, $q = cpl$	-0.0261	0.0034	0.0306	-0.0079
$s^p = cod$, $q = cpl$	-0.0246	-0.0060	0.0546	-0.0240
$s^p = co$, $q = cpl$	-0.0161	-0.0061	-0.0225	0.0447
Free College				
	$s = hsd$	$s = hs$	$s = cod$	$s = co$
$s^p = hsd$, $q = cpl$	-0.0011	-0.3091	0.2395	0.0706
$s^p = hs$, $q = cpl$	-0.0000	-0.2580	0.2245	0.0335
$s^p = cod$, $q = cpl$	-0.0074	-0.2684	0.2580	0.0178
$s^p = co$, $q = cpl$	-0.0043	-0.0083	0.0009	0.0117

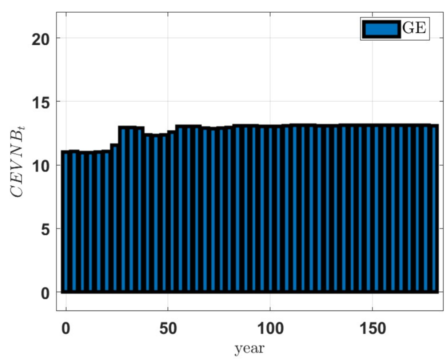
[32] The education level of parental households is determined by the highest educational degree obtained by either of the two parents.

A.4 The Importance of General Equilibrium for the Welfare Consequences of the Reform

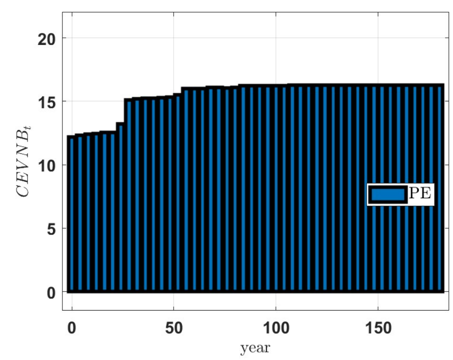
Stress the importance of the change in the interest rate for welfare effects. Wage movements induce negative GE effect since wage level per efficiency units fall and relative wages of college educated fall, thus absolute wages of college workers fall a lot, those of non college rise moderately (impact on inequality positive).

Main positive effect is (moderate) increase in interest rate, increases inter-vivos transfers and through it welfare. How do we know: if you keep inter vivos constant, most of the GE+r welfare gains relative to GE disappear.

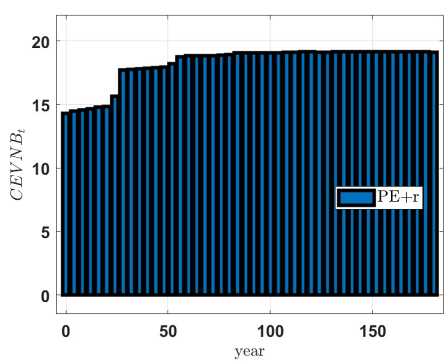
Figure 12: Free College Reform, Welfare Gains of Newborns



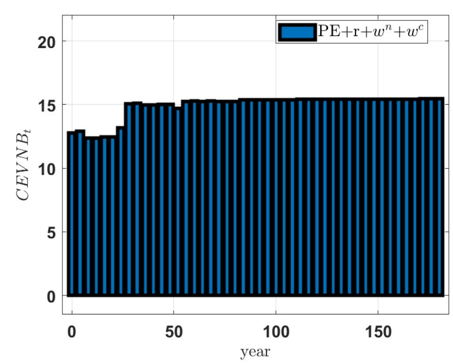
(a) CEV NB: GE



(b) CEV NB: PE

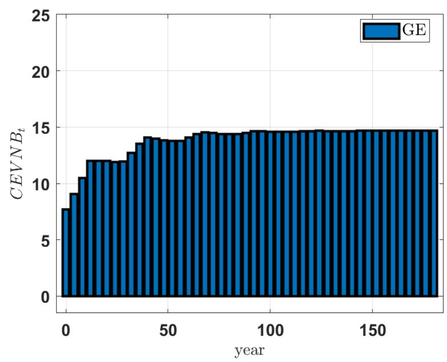


(c) CEV NB: PE+r

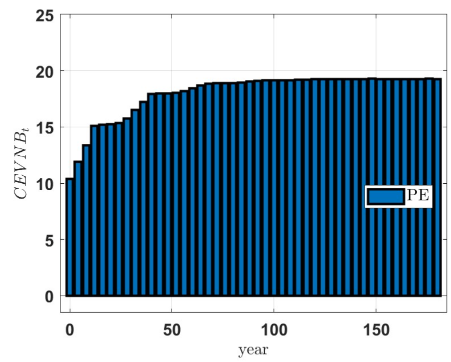


(d) CEV NB: PE+r+w^n+w^c

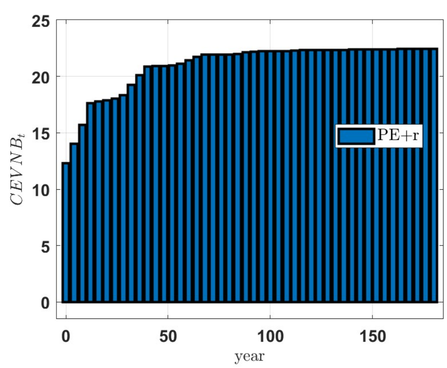
Figure 13: i^g Reform, Welfare Gains of Newborns



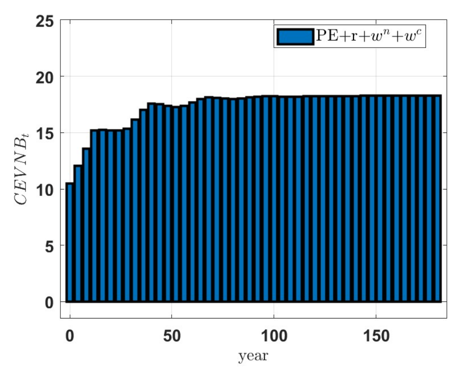
(a) CEV NB: GE



(b) CEV NB: PE

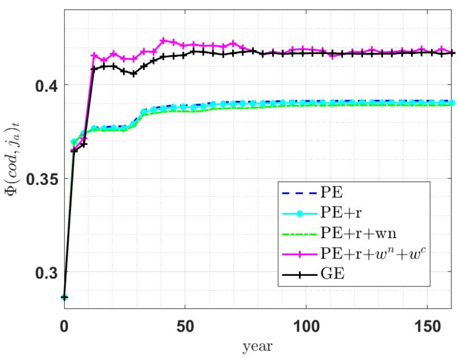


(c) CEV NB: PE+r

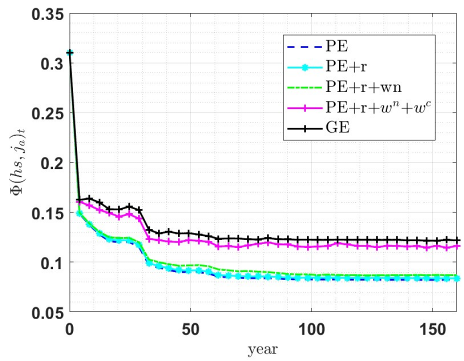


(d) CEV NB: PE+r+w^n+w^c

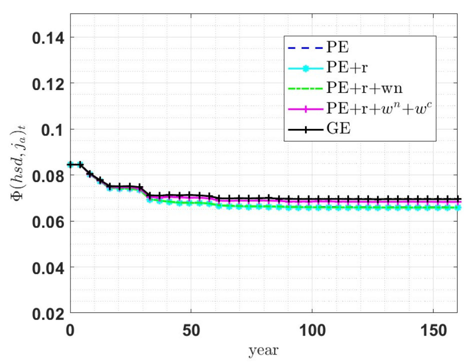
Figure 14: Mixed Reform, Education Shares



(a) College Dropout Share



(b) High School Share



(c) High School Dropout Share

B Model Appendix

B.1 Technological Progress and Population Growth

The level of labor-augmenting technological progress is denoted by Z_t which is assumed to evolve deterministically with a growth rate μ:

$$
\begin{aligned}
Z_t &= (1+\mu)Z_{t-1} && (34)\\
&= (1+\mu)^t Z_0 && (35)
\end{aligned}
$$

where Z_0 is normalized to 1.

The population growth n_t is allowed to be time-varying - under heterogeneous fertility rates the endogenous evolution of education shares in the transition results in the endogenous adjustments of the total population growth. Recall that all parents are assumed to give birth to children at the same age j_f with $\varsigma(s(wo))$ being education-specific fertility rates. Starting from retirement age j_r onwards households face non-zero survival risk with ϕ_j denoting the survival probability from age j to age $j+1$.

The population dynamics in every period evolve accordingly as

$$
\begin{aligned}
N_{t+1}(j_a) &= \sum_s N_t(j_a + j_f, s, wo) \cdot \varsigma(s(wo)) && (36)\\
N_{t+1}(j+1) &= N_t(j) \cdot \phi_j && (37)\\
N_{t+1} &= (1+n_t)N_t && (38)
\end{aligned}
$$

Throughout denote non-detrended aggregate variables by $\hat{X}_t$, and aggregate variables detrended by the rate of technological progress by X_t, i.e. $X_t = \frac{\hat{X}_t}{Z_t}$.

B.2 Production

Recall, the aggregate production function is given by:

$$\hat{Y}_t = F(\hat{K}_t, Z_t L_t) = \hat{K}_t^{\alpha}(Z_t L_t)^{1-\alpha} \quad (39)$$

Detrended by technology growth rate:

$$Y_t = K_t^{\alpha} L_t^{1-\alpha} \quad (40)$$

B.3 Aggregate Resource Constraint

In terms of non-detrended aggregate variables:

$$\hat{C}_t + \hat{K}_{t+1} + \hat{CE}_t + \hat{E}_t + \hat{G}_t = \hat{K}_t^{\alpha}(Z_t L_t)^{1-\alpha} + (1-\delta)\hat{K}_t \tag{41}$$

Detrended by technology and population growth rates (equivalent to expressing in per capita terms):

$$C_t + K_{t+1}(1+\mu_t)(1+n_t) + CE_t + E_t + G_t = K_t^{\alpha} L_t^{1-\alpha} + (1-\delta)K_t \tag{42}$$

B.4 Government Budget Constraints

General budget constraint in terms of non-detrended variables:

$$\hat{E}_t + \hat{E}_t^{CL} + \hat{G}_t + (1+r_t)\hat{B}_t = \hat{B}_{t+1} + \hat{T}_t + \tau_{c,t}\hat{C}_t + \tau_{k,t} r_t(\hat{K}_t + \hat{B}_t) \tag{43}$$

Detrended by technology and population growth rates (equivalent to expressing in per capita terms):

$$E_t + E_t^{CL} + G_t + (1+r_t)B_t = (1+\mu_t)(1+n_t)B_{t+1} + T_t + \tau_{c,t}C_t + \tau_{k,t} r_t(K_t + B_t) \tag{44}$$

Social security budget constraint in terms of non-detrended variables:

$$\tau_t^p(\hat{w}_{co,t}L_{co,t} + \hat{w}_{nc,t}L_{nc,t}) = \sum_{j=j_r}^{J} N_j \int \hat{pen}_t(s,\gamma,\eta)\, d\Phi_t \tag{45}$$

Detrended by technology growth rate:

$$\tau_t^p(w_{co,t}L_{co,t} + w_{nc,t}L_{nc,t}) = \sum_{j=j_r}^{J} N_j \int pen_t(s,\gamma,\eta)\, d\Phi_t \tag{46}$$

Observe that since both labor income taxes and social security benefits are non-homothetic in income, in the detrended versions both the labor income tax level parameter τ and the pension replacement rate ρ^p should be re-scaled accordingly.

B.5 Recursive Formulation of Household Problem

For illustration, below the dynamic problem of single mothers after children have left the household is presented in a non-detrended and detrended form, respectively.

The dynamic problem is given by:

$$V_t(j, si, wo, s, \gamma, \eta, \hat{a}) = \max_{\hat{c}, a', \ell} \{u(\hat{c}, \ell) - F(g)_{\ell>0} \\ + \beta \sum_{\eta'} \pi(\eta'|\eta) V_{t+1}(j+1, si, wo, s, \gamma, \eta', \hat{a}')\Big\}$$

subject to

$$\begin{aligned}
\hat{a}' + \hat{c}(1+\tau^c) + \hat{T}(\hat{y}(1-0.5\tau^p)) &= (\hat{a} + \hat{Tr}_{t,j})(1 + r(1-\tau^k)) + \hat{y}(1-\tau_t^p) \\
\hat{y} &= \hat{w}(s)\gamma(s)\epsilon(s, g, j)\eta\ell \\
\hat{a}' &\geq -\hat{\underline{\mathsf{a}}}(s, j) \\
\hat{c} &\geq 0 \\
\ell &\in [0, \Gamma^{si}].
\end{aligned}$$

Detrended version:

$$V_t(j, si, wo, s, \gamma, \eta, a) = \max_{c, a', \ell} \{u(c, \ell) - F(g)_{\ell>0} \\ + \beta \sum_{\eta'} \pi(\eta'|\eta) V_{t+1}(j+1, si, wo, s, \gamma, \eta', a')\Big\}$$

subject to

$$\begin{aligned}
a'(1+\mu) + c(1+\tau^c) + T(y(1-0.5\tau^p)) &= (a + Tr_{t,j})(1 + r(1-\tau^k)) + y(1-\tau^p) \\
y &= w(s)\gamma(s)\epsilon(s, g, j)\eta\ell \\
a' &\geq -\underline{\mathsf{a}}(s, j) \\
c &\geq 0 \\
\ell &\in [0, \Gamma^{si}].
\end{aligned}$$

B.6 Human Capital Production Function: Normalization

$$h' = \left(\kappa \left(\frac{i}{\bar{\bar{i}}} \right)^{\rho} + (1-\kappa) \left(\frac{l}{\bar{\bar{l}}} \right)^{\rho} \right)^{\frac{1}{\rho}}$$

Alternatively, this production function can be rewritten as

$$\begin{aligned} h' &= \left(\frac{\kappa}{\bar{\bar{i}}^{\rho}} i^{\rho} + \frac{(1-\kappa)}{\bar{l}^{\rho}} l^{\rho} \right)^{\frac{1}{\rho}} \\ &= \left(\Gamma \left(\frac{\frac{\kappa}{\bar{i}^{\rho}}}{\Gamma} i^{\rho} + \frac{\frac{(1-\kappa)}{\bar{l}^{\rho}}}{\Gamma} \right) l^{\rho} \right)^{\frac{1}{\rho}}, \end{aligned}$$

for $\Gamma(\kappa, \bar{i}, \bar{l}, \rho) \equiv \frac{\kappa}{\bar{i}^{\rho}} + \frac{(1-\kappa)}{\bar{l}^{\rho}}$ where, in general, $\Gamma(\cdot) \neq 1$. We can rewrite this further as

$$h' = \tilde{\Gamma} \left(\tilde{\kappa} i^{\rho} + (1-\tilde{\kappa}) l^{\rho} \right)^{\frac{1}{\rho}}$$

where $\tilde{\Gamma} = \Gamma^{\frac{1}{\rho}}$ and $\tilde{\kappa} = \frac{\frac{\kappa}{\bar{i}^{\rho}}}{\Gamma}$, and we note that $\tilde{\kappa} = \tilde{\kappa}(\kappa, \bar{i}, \bar{l}, \rho)$ and $\tilde{\Gamma} = \tilde{\Gamma}(\kappa, \bar{i}, \bar{l}, \rho)$.

B.7 Optimal Parental Human Capital Investments

Optimal Investment in Human Capital The intratemporal optimality condition for time with children reads as

$$\begin{aligned} \varsigma(si, s)(v^t)' \left(\varsigma(si, s) \cdot i^t \right) &= \lambda_i \frac{\partial i(i^m, i^t, i^g)}{\partial i^t} - \lambda_t \cdot \varsigma(si, s) = \lambda_h \frac{\partial g(h, i)}{\partial i} \frac{\partial i(i^m, i^t, i^g)}{\partial i^t} - \lambda_t \cdot \varsigma(si, s) \\ &= \beta \mathbf{E}_{\eta'|\eta} V_h(x'; a', h') \frac{\partial g(h, i)}{\partial i} \frac{\partial i(i^m, i^t, i^g)}{\partial i^t} \end{aligned}$$

The left hand side is the marginal cost of spending an additional time unit with children, the right hand side gives the discounted benefits, per child, of one additional unit of the final good being spent on education, where $\frac{\partial g(h,i)}{\partial i} \frac{\partial i(i^m,i^t,i^g)}{\partial i^m}$ is the marginal benefit of that spending on human capitall tomorrow, and $\mathbf{E}_{\eta'|\eta} V_h(x'; a', h')$ is the expected marginal benefit of a smarter child.

The intertemporal optimality condition for resource investment in children reads as

$$\begin{aligned}
\lambda_b \varsigma(si,s) &= \lambda_i \frac{\partial i(i^m,i^t,i^g)}{\partial i^m} = \lambda_h \frac{\partial g(h,i)}{\partial i}\frac{\partial i(i^m,i^t,i^g)}{\partial i^t} \\
\left(\beta \mathbf{E}_{\eta'|\eta} V_a(x';a',h') + \lambda_a\right)\varsigma(si,s) &= \beta \mathbf{E}_{\eta'|\eta} V_h(x';a',h') \frac{\partial g(h,i)}{\partial i}\frac{\partial i(i^m,i^t,i^g)}{\partial i^m} \\
\frac{u'\left(\frac{c}{1+\zeta_c\varsigma(si,s)}\right)}{1+\zeta_c\varsigma(si,s)}\frac{\varsigma(si,s)}{1+\tau_c} &= \beta \mathbf{E}_{\eta'|\eta} V_h(x';a',h') \frac{\partial g(h,i)}{\partial i}\frac{\partial i(i^m,i^t,i^g)}{\partial i^m}
\end{aligned}$$

The left hand side is the marginal cost of reducing spending on consumption goods by one unit, and the right hand side again gives the discounted per child benefits.

Optimal Allocation between Time and Money Taking the ratio between the first order conditions for time and money inputs yields

$$\begin{aligned}
\frac{(v^t)'\left(\varsigma(si,s)\cdot i^t\right)}{\beta \mathbf{E}_{\eta'|\eta} V_a(x';a',h') + \lambda_a} &= \frac{\frac{\partial i^p(j,i^m,i^t)}{\partial i^t}}{\frac{\partial i^p(j,i^m,i^t)}{\partial i^m}} \\
(1+\tau_c)\frac{(v^t)'\left(\varsigma(si,s)\cdot i^t\right)}{\frac{u'\left(\frac{c}{1+\zeta_c\varsigma(si,s)}\right)}{1+\zeta_c\varsigma(si,s)}} &= \frac{\frac{\partial i^p(j,i^m,i^t)}{\partial i^t}}{\frac{\partial i^p(j,i^m,i^t)}{\partial i^m}}
\end{aligned} \tag{47}$$

This equation simply states that the marginal rate of substitution between time and consumption times its relative price (the consumption tax rate) equals the marginal rate of transformation in the production of inputs for human capital production.

Using the functional forms for per-period utility and human capital production function, the relation between optimal resource and time investments can be written as

$$\begin{aligned}
\frac{i^m}{i^t} &= \left(\frac{1}{\chi}\kappa\left(\varsigma(si,s)\cdot i^t\right)^{\frac{1}{\psi}} c(1+\tau^c)\right)^{\sigma^m} \\
\Leftrightarrow \quad \frac{i^m}{(i^t)^{1+\frac{\sigma^m}{\psi}}} &= \left(\frac{1}{\chi}\kappa\left(\varsigma(si,s)\right)^{\frac{1}{\psi}} c(1+\tau^c)\right)^{\sigma^m}
\end{aligned} \tag{48}$$

B.8 Disentangling Efficiency and Redistribution when Measuring Welfare: The LSRA

To disentangle welfare benefits stemming from efficiency gains from those driven by redistribution, we use a wealth-based welfare criterion that follows the spirit of the lump-sum redistribution authority originally described in Auerbach and Kotlikoff (1987) and applied to a model with intragenerational heterogeneity in Nishiyama and Smetters (2005). Technically, our wealth-based measure is computed as follows. As a first step, individual-specific transfers are computed that would make the currently living households indifferent between the status quo and the reform scenario. These transfers are then aggregated up using the initial steady-state cross-sectional distribution and population shares. As a second step, for each newborn cohort in the transition and in the final steady state an ex-ante uniform transfer is computed that would make them indifferent between being born into the initial steady state or a given period of the transition (or a final steady state). Finally, a present discounted value (based on the market discount rate) of these ex-ante transfers is computed and added up with the aggregate transfer to the initially alive population.

Thus, the total transfer is given by:

$$W = \int \Psi_0(j,\cdot) d\Phi_0 + \sum_{t=0}^{\infty} \left(\frac{1}{1+r_0} \right)^t \Psi_t \tag{49}$$

For exposition purposes, we follow the approach in Kindermann and Krueger (2014) and express the resulting aggregate monetary transfer as an annuity C paying a constant consumption flow in every transition period and in the final steady state:

$$C \sum_{t=0}^{\infty} \left(\frac{1}{1+r_0} \right)^t = -W \tag{50}$$

Finally, we express the computed annuity value as percent of initial aggregate consumption:

$$\text{LSRA} = 100 \cdot \frac{C}{C_0} \tag{51}$$