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Simultaneous Signaling in an Oligopoly Model*

by

George J. Mailath

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Department of Economics
3718 Locust Walk
University of Pennsylvania
Philadelphia, PA 19104-6297

Abstract

This paper studies a two period linear model of differentiated oligopoly in which firms have private information about their costs and price is the strategic variable. In the first period of the unique separating equilibrium, firms charge prices above the level that would maximize first period expected profits. The separating equilibrium is contrasted with the information sharing benchmark in which cost data is revealed after the first period. Expected prices are lower and expected welfare is higher in the information sharing benchmark than in the separating equilibrium. The rankings that are obtained hold not only when the goods are substitutes, but surprisingly also when they are complements.

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1. Introduction

This paper studies a dynamic model of differentiated oligopoly in which all firms have private information about their costs and simultaneously signal their information by their pricing decisions.

The economics of one-sided signaling is, by now, well understood. As an illustration, consider limit pricing.[1] In this scenario, there is a monopolist with privately known costs choosing price (or quantity) and a potential entrant deciding on entry on the basis of the perceived profitability of that market. In (the separating or signaling) equilibrium, the monopolist charges his monopoly price if his costs are the highest possible and a price less than his monopoly price if his costs are not the highest possible, i.e., limit pricing occurs. The divergence from monopoly pricing is a result of the monopolist's incentives to understate the profitability of entry by understating his costs.

When there is more than one privately informed firm, each will still have incentives to understate costs (or overstate, depending on the structure of the market). However, simultaneous signaling generates a substantially different comparison of signaling behavior with behavior in the nonsignaling benchmark. Call the action that maximizes a firm's expected profits assuming the other firms do not condition their beliefs on the firm's actions, the myopic response. For example, in the limit pricing model this is just the monopoly price. In one-sided signaling, behavior in the nonsignaling benchmark is given by the myopic response. With simultaneous signaling, behavior in the nonsignaling benchmark is not the myopic response, since the myopic response is a response to the other

[1] The model is discussed in Salop [1979] and formally analyzed in Milgrom and Roberts [1982]. Mailath [1987] shows that there is only one separating equilibrium in the model with continuum types analyzed by Milgrom and Roberts [1982].

agents' signaling choices, not their nonsignaling choices.[2] As a result, comparisons with the nonsignaling benchmark become more delicate and more interesting.

An important and necessary assumption in this paper is that the range of possible costs is "small". This plays a role in two places: it ensures that every firm prefers to operate, and it facilitates the comparisons with the nonsignaling benchmark. A study of large ranges of possible costs would involve dealing with issues of exit, which are beyond the scope of this paper. Thus, this paper should be interpreted as a study of the impact of moderate degrees of private information on industry performance.

The next Section describes a two period linear model of differentiated oligopoly in which firms have private information about their costs and price is the strategic variable. The unique separating equilibrium path is derived in Section 3. On this signaling equilibrium path, in the first period firms charge prices above the level that would maximize first period expected profits. Section 4 compares the signaling equilibrium path with the nonsignaling benchmark in which cost data is revealed after the first period. Expected prices are lower and expected welfare is higher in the nonsignaling benchmark than in the signaling equilibrium path. The rankings that are obtained hold not only when the goods are substitutes, but surprisingly also when they are complements. Related issues and literature are discussed in Section 5. Section 6 concludes with a discussion of quantity competition and general demand.

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[2]A similar point arises in Harrington [1987] in the context of an entry deterrence oligopoly model with common costs.

2. The Model and Equilibrium Concept

The model has n firms selling differentiated products in each of two periods. In each period firms simultaneously choose prices. Unit costs of production are private information -- each firm knows his own costs but not the costs of any competitor. At the end of the first period, each firm observes the first period prices of the other firms and infers a range of unit costs for each of them. Each then chooses a second period price. A firm can attempt to influence his competitors' beliefs, and hence the prices charged in the second period, by appropriately choosing his first period price. If the goods are substitutes, a firm would like his competitors to charge high prices. All else equal, a competitor will charge a high price if he believes the firm has a high unit cost (since the firm would then charge a high price and the reaction curve has a positive slope). On the other hand, if the goods are complements, a firm would like his competitors to charge low prices. All else equal, a competitor will charge a low price if he, again, believes the firm has a high unit cost (since the reaction curve has a negative slope). In both cases, each firm (regardless of his true unit cost) has an incentive to be taken for a high cost firm by his competitors. In equilibrium, each firm understands that his competitor is facing these incentives and allows for them when inferring from first period prices.

Specifically, let θ_i be the i^{th} firm's constant unit cost, $i=1,\ldots,n$. Each θ_i is independently distributed on $[m_i,M_i]$, $m_i \geq 0$ with distribution function F_i. A strategy for firm i is a pair of measurable functions (τ_i^1,τ_i^2). The first function gives first period price as a function of unit cost, $\tau_i^1:[m_i,M_i] \rightarrow \Re_+$. The second function gives second period price as a function of unit cost and first period prices, $\tau_i^2 : [m_i, M_i] \times \Re_+^n \rightarrow \Re_+$. Since pure strategy equilibria exist, mixed strategies will not be discussed.

There is a continuum of identical consumers with quadratic utility function $U(q_1,..,q_n) + q_0$, where q_0 is the quantity of the numeraire good and q_i is the quantity of the i^{th} good. Consumers have the same utility function in each period. Letting p_i be the price of the i^{th} good, the representative consumer maximizes consumer surplus:[3]

$$CS = U(q_1,\ldots,q_n) - p'q$$
$$= (\alpha - p)'q - \tfrac{1}{2}q'Bq,$$

where $\alpha_i > 0$ and B is a positive definite symmetric matrix. The vector of (aggregate) quantities maximizing CS subject to $q_i \geq 0$ is denoted q(p). When prices result in an interior maximum, the vector of (aggregate) quantities is $q(p) = B^{-1}(\alpha - p) \equiv C(\alpha - p)$. Since the utility function has no income effects, these demands are both the compensated (Hicksian) and uncompensated (Marshallian) demands. Goods i and j are substitutes if $c_{ij} < 0$ and complements if $c_{ij} > 0$ (where c_{ij} is the ij^{th} element of C, the inverse of B).

It is necessary to make an assumption to ensure that in both periods in equilibrium strictly positive quantities are demanded. Quantities in equilibrium will be positive in the second period and prices will exceed marginal cost if $C(C+\Delta(C))^{-1}\Delta(C)(\alpha - \theta) >> 0$,[4] for all θ, where $\Delta(C)$ is the diagonal matrix with c_{ii} for the ii^{th} element and zero elsewhere. Suppose $m \equiv (m_1,..,m_n)'$, α and C satisfy

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[3]The notational conventions are as follows: unsubscripted variables are vectors of the corresponding firm variables, $x_{-i} \equiv (x_1,..,x_{i-1},x_{i+1},..,x_n)$, and $(x_i', x_{-i}) \equiv (x_1,..,x_{i-1},x_i',x_{i+1},..,x_n)$.

[4]For two vectors, x and y, $x >> y$ iff $x_i > y_i$, $\forall i$. The one-shot game where the firms' unit costs are common knowledge, firm i chooses p_i and receives a payoff of $(p_i - \theta_i)q_i(p_1,..,p_n)$ has a unique Nash equilibrium given by $\xi(\theta) \equiv (C+\Delta(C))^{-1}(C\alpha + \Delta(C)\theta)$. The inequality in the text follows from $C(\alpha - \xi(\theta)) >> 0$.

$C(C+\Delta(C))^{-1}\Delta(C)(\alpha-m) \gg 0$. If M_i-m_i is not too large for each i, then strictly positive quantities are also demanded in the first period in the separating equilibrium.[5] Rather than explicitly calculating bounds on M_i-m_i (which is tedious and not illuminating), it will simply be assumed that M_i-m_i is not too large.

<u>Assumption</u>: The parameters m, α, and C satisfy $C(C+\Delta(C))^{-1}\Delta(C)(\alpha-m) \gg 0$, and M_i-m_i are not too large for all i, so that in equilibrium positive quantities are demanded in both periods.

Firm i is risk neutral and the discounted sum of profits is given by $\Pi_i(p^1,p^2,\theta_i) = (p_i^1-\theta_i)q_i(p^1)+\delta(p_i^2-\theta_i)q_i(p^2)$, $0<\delta<1$. Suppose firm j is playing (τ_j^1,τ_j^2), $j\neq i$. Firm i's best response to the strategies of the other firms maximizes his expected discounted sum of profits. At the beginning of the second period, the firms have observed the vector of first period prices, p^1. Firm i then knows that j's unit cost lies in the set $B_j \equiv (\tau_j^1)^{-1}(p_j^1)$. In particular, if j's first period strategy is one-to-one, this set is a singleton and i (as well as the other firms) knows j's unit cost exactly. Thus, (τ_i^1,τ_i^2) is a best response if $\tau_i^1(\theta_i)$ is maximizing, i.e.,

$$(1) \qquad \tau_i^1(\theta_i) \in \underset{p_i}{\text{argmax}}\ E\{(p_i-\theta_i)q_i(p_i,\tau_{-i}^1(\theta_{-i})) + \delta P_i^2(\theta_i,p_i,\tau_{-i}^1(\theta_{-i}))\},$$

where $P_i^2(\theta_i,p^1)$ is the maximized value of second period expected profits, conditional on $\theta_j\in B_j$ for $j\neq i$, i.e.,

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[5]In the separating equilibrium, a firm's first period price is continuous and increasing in unit cost, and the lowest cost firm maximizes first period expected profits (see Theorem 1 below).

(2) $$P_i^2(\theta_i,p^1) = \max_{p_i} E\{(p_i-\theta_i)q_i(p_i,\tau_{-i}^2(\theta_{-i},p^1))\,|\,\theta_j\in B_j, j\neq i\},$$

for $p_j^1\in\tau_j^1([m_j,M_j])$, $j\neq i$, and if $\tau_i^2(\theta_i,p^1)$ is maximizing for appropriate p^1, i.e.,

(3) $$\tau_i^2(\theta_i,p^1) \in \operatorname*{argmax}_{p_i} E\{(p_i-\theta_i)q_i(p_i,\tau_{-i}^2(\theta_{-i},p^1))\,|\,\theta_j\in B_j, j\neq i\},$$

for $p_j^1\in\tau_j^1([m_j,M_j])$, $j\neq i$, and $p_i^1=\tau_i^1(\theta_i)$.

A strategy profile $(\tau^1,\tau^2) = \{(\tau_1^1,\tau_1^2),\ldots,(\tau_n^1,\tau_n^2)\}$ is a <u>Bayesian Nash equilibrium</u> if (τ_i^1,τ_i^2) is a best response to $(\tau_{-i}^1,\tau_{-i}^2)$, for all i.

It is necessary to impose a credibility restriction on the behavior of firms in Bayesian Nash equilibria when there is a detectable deviation from the equilibrium. In particular, it will be required that the strategy profile be a sequential equilibrium (Kreps and Wilson [1982]).[6] In such an equilibrium firms always choose prices optimally in the second period with respect to some beliefs that do not contradict the common knowledge structure of the game. Given τ^1 and p^1, the firms can be partitioned into two groups. One group, $N(p^1)$, consists of those firms for whom the first period price is in the image of their first period strategy, i.e., $N(p^1) = \{i:p_i^1\in\tau_i^1([m_i,M_i])\}$. The other group, $N^c(p^1)$, consists of those firms who have made a detectable deviation, i.e., $N^c(p^1) = \{i:p_i^1\notin\tau_i^1([m_i,M_i])\}$. Only when $N^c(p^1)$ is empty will Bayes' rule completely determine the firms' beliefs about θ.[7] Beliefs need to be specified for any p^1 for which $N^c(p^1)$ is nonempty. These beliefs are assumed to be the same for all the firms (this is an immediate implication of sequentiality and independence).

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[6]While Kreps and Wilson are concerned only with finite games, there is an obvious extension of their concept to this game.

[7]I am ignoring the fact that conditional probabilities are only unique almost everywhere.

The appropriate restrictions on beliefs in this game is merely that the support of these beliefs lie in the supports of the F_i and that the belief about θ_i, $i \in N(p^1)$, should equal the prior distribution updated by $\theta_i \in (\tau_i^1)^{-1}(p_i^1)$. More formally:

Definition: The strategy profile (τ^1, τ^2) is a sequential equilibrium if

(i) (τ^1, τ^2) is a Bayesian Nash equilibrium; and

(ii) there exists for each j a distribution function H_j on $[m_j, M_j]$ such that for all i, for all θ_i, $\tau_i^2(\theta_i, p^1)$ maximizes

$$\int \cdot \cdot \int (p_i - \theta_i) q(p_i, \tau_{-i}^2(\theta_{-i}, p^1)) \; \Pi_{j \neq i} H_j(d\theta_j),$$

where $H_j(.) = F_j(. | \theta_j \in (\tau_j^1)^{-1}(p_j^1))$ for any $j \in N(p^1)$, $j \neq i$.

Equilibria fall into two groups. The first group are the separating equilibria, in which each firm's first period strategy is a one-to-one function. In a separating equilibrium the private information is completely revealed by first period prices and in the second period firms are essentially playing a one-shot game of complete information. The nonseparating, or semipooling, equilibria involve varying degrees of pooling (different types charging the same price).

3. The Separating Equilibrium

While this game has many sequential equilibria, there is essentially only one in which each firm is separating. In order to be more specific, one more definition is required. An outcome path in this game is, for each type of each firm, a specification of a first period price and a contingent second period price, where the set of contingencies are the set of possible first period prices of the other firms that can result from the specified first period prices. An outcome path induced by an equilibrium is an equilibrium path. Two profiles, (τ^1,τ^2) and $(\underline{\tau}^1,\underline{\tau}^2)$, induce the same outcome path if $\tau_i^1=\underline{\tau}_i^1$ for all i and $\tau_i^2(\theta_i,\tau_i^1(\theta_i),p_{-i}^1)=\underline{\tau}_i^2(\theta_i,\tau_i^1(\theta_i),p_{-i}^1)$ for all $p_{-i}^1\in\Pi_{j\neq i}\tau_j^1([m_j,M_j])$. In other words, two profiles induce the same outcome path if following the two profiles yields the same pattern of prices in the two periods.

Let $\phi_i(\tau_{-i}^1,\theta_i)$ be the price which maximizes firm i's first period expected profits when the other firms are playing τ_{-i}^1 (this is the myopic response discussed in the Introduction). That is,[8]

$$\phi_i(\tau_{-i}^1,\theta_i) = \{C_{i\bullet}\alpha - \Sigma_{j\neq i}c_{ij}E\tau_j^1(\theta_j)\}(2c_{ii})^{-1} + \tfrac{1}{2}\theta_i. \tag{4}$$

The incentives for a firm to be taken for a high cost type by his competitor result in separating equilibrium prices exceeding first period expected profit maximizing prices (except for the least cost type). The characterization of the unique separating sequential equilibrium path is contained in the following theorem.

<u>Theorem 1</u>: There exists a separating sequential equilibrium, $(\hat{\tau}^1,\hat{\tau}^2)$. All separating sequential equilibria induce the same equilibrium path, which has the following properties:

(i) $\hat{\tau}_i^1(m_i) = \phi_i(\hat{\tau}_{-i}^1,m_i)$;

(ii) $d\hat{\tau}_i^1/d\theta_i > 0$ for all $\theta_i\in(m_i,M_i]$;[9]

(iii) $\hat{\tau}_i^1(\theta_i) > \phi_i(\hat{\tau}_{-i}^1,\theta_i)$ for all $\theta_i\in(m_i,M_i]$; and

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[8] $C_{i\bullet}$ is the i^{th} row of the matrix C.

[9] The function $\hat{\tau}_i^1$ has infinite slope at m_i.

(iv) for all N, there exists M_i such that $d\hat{\tau}_i^1(\theta_i)/d\theta_i > N$, $\forall\theta_i \in (m_i, M_i]$.[10]

<u>Proof</u>: First some necessary conditions for a profile to be a separating sequential equilibrium are derived. Suppose (τ^1, τ^2) is a separating strategy profile (so that each agent's first period strategy is one-to-one). In order for this profile to be a Bayesian Nash equilibrium it is necessary that $(\tau_1^2(\theta_1, \tau^1(\theta)), .., \tau_n^2(\theta_n, \tau^1(\theta)))$ be a Nash equilibrium of the second period subform considered as a one-shot game where the firms' unit costs are common knowledge, i.e., firm i chooses p_i^2 and receives a payoff of $(p_i^2 - \theta_i)q_i(p^2)$. This one-shot game has a unique Nash equilibrium, $\xi(\theta)$, given by $\xi(\theta) \equiv (C+\Delta(C))^{-1}(C\alpha+\Delta(C)\theta)$.

Suppose firm i chooses $p_i^1 \in \tau_i^1([m_i, M_i])$ in the first period and the other firms follow τ_{-i}^1. The other firms will infer that his cost is $\hat{\theta}_i = (\tau_i^1)^{-1}(p_i^1)$ and behave accordingly. Apart from firm i, unit costs have been correctly inferred and in the second period the firms other than i will charge according to $\xi_{-i}(\hat{\theta}_i, \theta_{-i})$. In the second period firm i chooses p_i^2 to maximize $(p_i^2 - \theta_i)q_i(p_i^2, \xi_{-i}(\hat{\theta}_i, \theta_{-i}))$; let $\xi_i^*(\hat{\theta}_i, \theta_i, \theta_{-i})$ be the maximizing price:

(5) $$\xi_i^*(\hat{\theta}_i, \theta_i, \theta_{-i}) \equiv \{C_{i\bullet}\alpha - \Sigma_{j\neq i} c_{ij}\xi_j(\hat{\theta}_i, \theta_{-i})\}(2c_{ii})^{-1} + \tfrac{1}{2}\theta_i.$$

Firm i's reduced form expected discounted profits, given τ_{-i}^1 and second period optimal prices for the other firms and after substituting (5), is

(6) $$\Xi_i(p_i^1, \hat{\theta}_i, \theta_i) \equiv E\{(p_i^1 - \theta_i)q_i(p_i^1, \tau_{-i}^1(\theta_{-i})) + \delta c_{ii}(\xi_i^*(\hat{\theta}_i, \theta_i, \theta_{-i}) - \theta_i)^2\}.$$

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[10] This property is used in the next Section.

The payoff to firm i of choosing $p_i^1 \in \tau_i^1([m_i,M_i])$ is $\Xi_i(p_i^1,(\tau_i^1)^{-1}(p_i^1),\theta_i)$. For τ_i^1 to be a best response for firm i, $\tau_i^1(\theta_i)$ must maximize $\Xi_i(p_i^1,(\tau_i^1)^{-1}(p_i^1),\theta_i)$. So for (τ^1,τ^2) to be a separating equilibrium, $\tau_i^1(\theta_i)$ must be optimal for firm i with cost θ_i over $p_i^1 \in \tau_i^1([m_i,M_i])$. This <u>incentive compatibility</u> requirement has strong implications. By Theorem 2 of Mailath [1987],[11] τ_i^1 is increasing, continuous on $[m_i,M_i]$, differentiable on $(m_i,M_i]$ and so satisfies

$$d\tau_i^1/d\theta_i = \frac{-\partial\Xi_i(\tau_i^1(\theta_i),\theta_i,\theta_i)/\partial\hat{\theta}_i}{\partial\Xi_i(\tau_i^1(\theta_i),\theta_i,\theta_i)/\partial p_i^1},$$

for $\theta_i \in (m_i,M_i]$. It is immediate that $\partial\Xi_i(p_i^1,\hat{\theta}_i,\theta_i)/\partial p_i^1 = 2c_{ii}[\phi_i(\tau_{-i}^1,\theta_i) - p_i^1]$. Now, $\partial\Xi_i(p_i^1,\hat{\theta}_i,\theta_i)/\partial\hat{\theta}_i = 2\delta c_{ii}E(\xi_i^*(\hat{\theta}_i,\theta_i,\theta_{-i})-\theta_i)\partial\xi_i^*(\hat{\theta}_i,\theta_i,\theta_{-i})/\partial\hat{\theta}_i$. Since $\xi_i^*(\hat{\theta}_i,\theta_i,\theta_{-i}) = \xi_i(\hat{\theta}_i,\theta_{-i}) - ½(\hat{\theta}_i-\theta_i)$, $\partial\xi_i^*(\hat{\theta}_i,\theta_i,\theta_{-i})/\partial\hat{\theta}_i = \partial\xi_i(\hat{\theta}_i,\theta_{-i})/\partial\hat{\theta}_i - ½ = c_{ii}[(C+\Delta(C))^{-1}]_{ii} - ½$. The Appendix contains a proof that $\partial\Xi_i(p_i^1,\theta_i,\theta_i)/\partial\hat{\theta}_i > 0$. Since ξ is linear, the differential equation can be written as

$$d\tau_i^1/d\theta_i = h_i(\theta_i,E\theta_{-i})/(\tau_i^1(\theta_i) - \phi_i(\tau_{-i}^1,\theta_i)), \tag{7}$$

where $h_i(.,.)$ is a positive linear function. This implies $\tau_i^1(\theta_i) > \phi_i(\tau_{-i}^1,\theta_i)$ for $\theta_i \neq m_i$. Since the worst inference about firm i's unit cost is that it equals m_i, m_i cannot be punished for any deviation in a separating sequential equilibrium,

[11]The model here violates one of the assumptions of Mailath [1987], namely that $\partial\Xi_i/\partial\hat{\theta}_i$ never equal zero on the domain. However, since the initial value condition is satisfied, a study of the proof reveals that it is only necessary that $\partial\Xi_i/\partial\hat{\theta}_i$ never equal zero for $\hat{\theta}_i$ close to θ_i.

and so sequentiality requires $\tau_i^1(m_i) = \phi_i(\tau_{-i}^1, m_i)$ (see the Proposition in the Appendix).[12]

The restricted initial value problem given by (7), $\tau_i^1(m_i) = \phi_i(\tau_{-i}^1, m_i)$ and $d\tau_i^1/d\theta_i > 0$ has a unique solution for all τ_{-i}^1 (Mailath [1987, Proposition 5]). Furthermore, if the implied prices always yield strictly positive demand for good i, then the solution satisfies the incentive compatibility requirement for firm i (Mailath [1987, Theorem 3]).

The expected value of τ^1 must now be determined. Let τ_i^0 be the (unique) increasing solution to the initial value problem $(\tau_i^1 - \phi_i(\mathbf{0},\theta_i))d\tau_i^1/d\theta_i = h_i(\theta_i, E\theta_{-i})$, $\tau_i^1(m_i) = \phi_i(\mathbf{0}, m_i)$, where $\mathbf{0}$ is the zero function. The profile τ^1 simultaneously solves all of the restricted initial value problems if, and only if, $\tau_i^0(\theta_i) = \tau_i^1(\theta_i) + (2c_{ii})^{-1}\Sigma_{j\neq i}c_{ij}E\tau_j^1(\theta_j)$. [This follows from $\phi_i(\tau_{-i}^1, \theta_i) = \phi_i(\mathbf{0}, \theta_i) - (2c_{ii})^{-1}\Sigma_{j\neq i}c_{ij}E\tau_j^1(\theta_j)$.] Solving gives a unique value for $E\tau^1(\theta)$, namely $2(C+\Delta(C))^{-1}\Delta(C)E\tau^0$. The strategy $\hat{\tau}_i^1$ can now be calculated from the restricted initial value problem, since ϕ_i only depends on τ_{-i}^1 through its expected value. The last property of $\hat{\tau}^1$ follows immediately from the fact that $d\hat{\tau}_i^1/d\theta_i = d\tau_i^0/d\theta_i$.

To complete the proof it is only necessary to provide the off-the-equilibrium path beliefs which support $\hat{\tau}^1$. For $p_i^1 < \hat{\tau}_i^1(m_i)$, firms $j\neq i$ believe i's cost is equal to m_i and for $p_i^1 > \hat{\tau}_i^1(M_i)$, firms $j\neq i$ believe i's cost is equal to M_i. <u>QED</u>

[12] Observe that a firm's first period strategy in the separating sequential equilibrium must have an unbounded derivative, since it must satisfy the differential equation (7) (which is implied by Nash) and the initial value condition $\hat{\tau}_i^1(m_i) = \phi_i(\hat{\tau}_{-i}^1, m_i)$ (which is implied by sequentiality).

To emphasize the nature of behavior on the unique separating equilibrium path, I will refer to it as the signaling equilibrium path, or more simply (with a slight abuse of terminology) the signaling equilibrium.

As was noted earlier, this game also has many (in fact, a continuum of) semipooling equilibria.[13] The source of the multiplicity is the same as in other signaling games -- there are many ways in which inferences about the private information can be formed from observed actions. The signaling equilibrium path is the natural equilibrium path to study for several reasons. It is the equilibrium path that maximizes the amount of information transmitted (in the second period, all the firms are completely informed about the cost structures of their competitors), and so it most clearly demonstrates the strategic nature of the information transmission. It can also be argued that it is the most salient equilibrium path because it is the only separating equilibrium path. The signaling equilibrium path is also the equilibrium path usually studied in signaling models (for example, Milgrom and Roberts [1982]).

A common response to the multiplicity of sequential equilibria in standard one-sided signaling games has been to eliminate equilibria using the various refinements related to (inspired by) the notion of strategic stability due to Kohlberg and Mertens [1986]. The details of this are admirably discussed in Cho and Kreps [1987], Banks and Sobel [1987] and Cho and Sobel [1987]. In particular, Cho and Sobel [1987] provide conditions on one-sided signaling games such that the Pareto dominating (for the informed agent) separating sequential equilibrium path is the unique stable outcome. While the results of Cho and

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[13]The semipooling equilibria of a related game are detailed in Mailath [1985b]. That paper analyzes a one-sided signaling model whose signaling structure is very similar to the one here. The simultaneity does not affect this portion of the analysis.

Sobel [1987] are not directly applicable, I conjecture that a similar result could be proved here.[14] Zachau [1986] proves that the separating sequential equilibrium path is the only stable outcome path when there are only two firms and only two possible costs for each firm (rather than a continuum or an arbitrary finite number). Caminal [1988] proves a similar result when the private information is about demand (again, the private information takes only two possible values).

4. Comparison with the Nonsignaling Benchmarks

I now consider how simultaneous signaling affects firm behavior and welfare. There are two possible nonsignaling benchmarks. The first nonsignaling benchmark is given by the equilibrium of the game in which all the cost information becomes -- for some reason -- common knowledge at the end of the first period. In this game the impact of first period prices on second period behavior is eliminated (and so there is no signaling), but the differential information of the first period is preserved. The first benchmark, which I will now refer to as the <u>interim benchmark</u>, is the unique equilibrium of this game. The second benchmark is the <u>ex ante benchmark</u>, in which cost data sharing occurs before the first period. In the ex ante benchmark, firms are always playing a game of complete information. Since the information available to the firms at the beginning of the second period is identical in the three situations, and this

[14]Strategic stability is only defined for finite games. Cho and Sobel [1987] analyze signaling games with a continuum of actions (the restrictions on beliefs they impose are implied by stability). This game also has a continuum of types. The proof would require dealing with discretizations of the type spaces.

second period one-shot game has a unique equilibrium, second period behavior is identical. The welfare comparisons need only be done for the first period.

For this author, the most interesting and relevant benchmark is the interim benchmark. It involves the smallest change in information structure to remove signaling and so most clearly illustrates the nature of the interactions that occur with simultaneous signaling. The welfare comparisons that are discussed below assume that society has the capability of enforcing a regime in which firms honestly disclose their costs. Consider how the regulations which would enforce such a regime are enacted. If the source of the regulations is Congress, a bill is introduced, voted on and signed by the President. If the source of the regulations is an agency such as the Federal Trades Commission, the regulations are first published in the Federal Register, after which there is time for comments and hearings, as well as possible judicial review. In general, regulations are not (and cannot be) retroactive. Thus any regulations will be public knowledge before they take effect. Firms would be competing in an environment where they know costs will be revealed at the end of the current period, but not at the beginning.

The welfare measure adopted in this paper is the sum of consumer surplus and total profits. If p is a vector of prices, then welfare is (after simplifying and multiplying by 2) given by

$$W(p,\theta) = (\alpha-\theta)'C(\alpha-\theta) - (p-\theta)'C(p-\theta). \tag{8}$$

The pricing rule which maximizes welfare under complete information is, as usual, marginal cost pricing.

Consider now the interim benchmark where publication of cost data occurs at the end of the first period. Let λ be the vector of first period equilibrium benchmark strategies:

$$(9) \qquad \lambda_i(\theta_i) = \{C_{i\bullet}\alpha - \Sigma_{j\neq i}c_{ij}E\lambda_j(\theta_j)\}(2c_{ii})^{-1} + \tfrac{1}{2}\theta_i,$$

where $E\lambda(\theta) = (C+\Delta(C))^{-1}(C\alpha+\Delta(C)E\theta)$. Note that $\lambda_i(\theta_i)=\phi_i(\lambda_{-i},\theta_i)$.

If all the goods are substitutes (i.e., $c_{ij}<0$ for all $i\neq j$), then prices in the signaling equilibrium are higher than in the interim benchmark for every realization of costs. The upward pressure on prices due to the incentives to exaggerate costs is reinforced by the positively sloped first period reaction curve. As long as the range of possible costs is not too large welfare is higher in the interim benchmark than in the signaling equilibrium for every realization.

Theorem 2: Suppose all goods are substitutes, i.e., $c_{ij}<0$ $\forall i\neq j$.

(i) For all realizations of unit costs, prices are higher in the signaling equilibrium than in the interim benchmark, i.e., $\hat{\tau}^1(\theta) \gg \lambda(\theta)$ $\forall\theta$.

(ii) If the quantities demanded under marginal cost prices exceed the quantities demanded under interim benchmark prices (i.e., $C(\alpha-\theta) \gg C(\alpha-\lambda(\theta))$), then ex post welfare is higher in the interim benchmark than in the signaling equilibrium.

(iii) Quantities traded need not always be less in the signaling equilibrium than in the interim benchmark.

Proof: (i) Let $y(\theta) = \hat{\tau}^1(\theta)-\lambda(\theta)$. Then for $\theta_i\neq m_i$, $y_i(\theta)>\phi_i(\hat{\tau}^1_{-i},\theta_i)-\phi_i(\lambda_{-i},\theta_i)$ $= (2c_{ii})^{-1}\Sigma_{j\neq i}c_{ij}E(\lambda_j(\theta_j)-\hat{\tau}^1_j(\theta_j))$. Taking expectations and simplifying gives

$(C+\Delta(C))Ey(\theta) \gg 0$. Since $c_{ij}<0$ for $i\neq j$, every element of $(C+\Delta(C))^{-1}$ is nonnegative.[15] Thus, $Ey(\theta) \gg 0$ and so $y(\theta) \gg 0$.

(ii) Ex post welfare is higher if $(\hat{\tau}^1(\theta)-\theta)'C(\hat{\tau}^1(\theta)-\theta) > (\lambda(\theta)-\theta)'C(\lambda(\theta)-\theta)$. Let $u(\theta) = \hat{\tau}^1(\theta)-\theta$ and $v(\theta) = \lambda(\theta)-\theta$. Now, $0 \leq (u(\theta)-v(\theta))'C(u(\theta)-v(\theta)) = u(\theta)'Cu(\theta)-2u(\theta)'Cv(\theta)+v(\theta)'Cv(\theta) < u(\theta)'Cu(\theta)-v(\theta)'Cv(\theta)$ [since $y(\theta) \gg 0$ and $Cv(\theta) \gg 0$, $y(\theta)'Cv(\theta) > 0$].

(iii) Suppose there are two symmetric firms (i.e., $n=2$, $c_{11}=c_{22}$, $m_1=m_2$, $M_1=M_2$ and $f_1=f_2$, so $\lambda_1=\lambda_2$, $\hat{\tau}^1_1=\hat{\tau}^1_2$) and that M_1 is such that $d\hat{\tau}^1_1/d\theta_1>\frac{1}{2}$. Consider the realization (m_1,M_1), i.e., firm 1 has cost m_1 and firm 2 has cost M_1. The quantity of good 1 sold in the signaling equilibrium will exceed that sold in the interim benchmark if $-c_{12}(\hat{\tau}^1_1(M_1)-\lambda_1(M_1)) > c_{11}(\hat{\tau}^1_1(m_1)-\lambda_1(m_1)) = -c_{12}(E\hat{\tau}^1_1(\theta)-E\lambda_1(\theta))/2$. However, $d\hat{\tau}^1_1/d\theta_1 > \frac{1}{2}$ ensures that $\hat{\tau}^1_1(\theta)-\lambda_1(\theta)$ is an increasing function and so $\hat{\tau}^1_1(M_1)-\lambda_1(M_1) > E\hat{\tau}^1_1(\theta)-E\lambda_1(\theta)$. QED

The fact that prices are higher in the signaling equilibrium when all the goods are substitutes is not sufficient reason to conclude that welfare is lower. An isowelfare locus for the two firm case is illustrated in Figure 1. The price vector p^* generates lower welfare than p^0, even though $p^* \ll p^0$. It is important to recognize the role of the substitution matrix C in weighting divergences of different products' prices. While p^* is closer to θ coordinate-wise than p^0, its divergence from marginal cost pricing as measured by the norm induced by C, i.e., $\|x\|_C \equiv \sqrt{(x'Cx)}$, is greater. The quantities demanded under marginal cost prices will exceed quantities demanded under interim benchmark prices if the range of possible costs is not too large. Parts (ii) and (iii) of the Theorem are not

[15] Use induction and the formula for a partitioned inverse.

in conflict -- welfare in the example will be lower because so little of good 2 is sold.

For the case of complements ($c_{ij}>0$ for all $i\neq j$), even though prices in the signaling equilibrium are higher than the first period profit maximizing level, they are <u>not</u> necessarily higher than the prices in the interim benchmark. When the goods are complements, a firm's reaction function, ϕ_i, is <u>decreasing</u> in the vector of competitors' expected prices. Suppose for a moment the firms are symmetric. Then expected prices are higher in the signaling equilibrium.[16] The realization in which each firm has cost m_1 results in a signaling equilibrium price of $\hat{\tau}_1^1(m_1)$ for each firm and $\hat{\tau}_1^1(m_1) = \phi_1(\hat{\tau}_1^1, m_1) < \phi_1(\lambda_1, m_1) = \lambda_1(m_1)$. Since $\hat{\tau}_1^1$ and λ_1 are continuous, there is therefore a positive probability that prices for <u>all</u> firms in the signaling equilibrium will be lower than in the interim benchmark. The higher expected price of the competitor exerts a downward pressure on a firm's price, which for low cost types dominates the upward pressure from the incentive to exaggerate costs. Further, since for complements lower prices unambiguously mean higher welfare, there is a positive probability that ex post welfare is higher in the signaling benchmark. However, if expected prices in the signaling equilibrium exceed those in the interim benchmark (as they do when the firms are symmetric) and if the range of costs is not large, then expected welfare is lower in the signaling equilibrium.

<u>Theorem 3</u>: Suppose all goods are complements, i.e. $c_{ij}>0$ $\forall i\neq j$. In expected terms, welfare in the signaling equilibrium is less than in the interim benchmark, if the following hold:

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[16]If $E\hat{\tau}_1^1(\theta_1) \leq E\lambda_1(\theta_1)$, then $\hat{\tau}_1^1(\theta_1) > \phi_1(\hat{\tau}_1^1, \theta_1) \geq \phi_1(\lambda_1, \theta_1) = \lambda_1(\theta_1)$ which is a contradiction.

(i) $E\hat{\tau}_i^1(\theta_i) > E\lambda_i(\theta_i)\ \forall i$; and

(ii) $d\hat{\tau}_i^1(\theta_i)/d\theta_i > 3/2\ \forall \theta_i \in (m_i, M_i],\ \forall i$.

<u>Proof</u>: The signaling equilibrium has lower expected welfare if, in addition to $E\hat{\tau}_i^1(\theta_i) > E\lambda_i(\theta_i)\ \forall i$, $E\{(\hat{\tau}_i^1(\theta_i)-\theta_i)^2\} > E\{(\lambda_i(\theta_i)-\theta_i)^2\}$. Now,

$$\int [(\hat{\tau}_i^1(\theta_i)-\theta_i)^2 - (\lambda_i(\theta_i)-\theta_i)^2]\, dF_i(\theta_i)$$

$$= \int_{\{\hat{\tau}_i^1(\theta_i)<\lambda_i(\theta_i)\}} (\hat{\tau}_i^1(\theta_i)+\lambda_i(\theta_i)-2\theta_i)(\hat{\tau}_i^1(\theta_i)-\lambda_i(\theta_i))\, dF_i(\theta_i)$$

$$+ \int_{\{\hat{\tau}_i^1(\theta_i)\geq\lambda_i(\theta_i)\}} (\hat{\tau}_i^1(\theta_i)+\lambda_i(\theta_i)-2\theta_i)(\hat{\tau}_i^1(\theta_i)-\lambda_i(\theta_i))\, dF_i(\theta_i)$$

$$> \sup_{\{\hat{\tau}_i^1(\theta_i)<\lambda_i(\theta_i)\}} (\hat{\tau}_i^1(\theta_i)+\lambda_i(\theta_i)-2\theta_i) \int_{\{\hat{\tau}_i^1(\theta_i)<\lambda_i(\theta_i)\}} (\hat{\tau}_i^1(\theta_i)-\lambda_i(\theta_i))\, dF_i(\theta_i)$$

$$+ \inf_{\{\hat{\tau}_i^1(\theta_i)\geq\lambda_i(\theta_i)\}} (\hat{\tau}_i^1(\theta_i)+\lambda_i(\theta_i)-2\theta_i) \int_{\{\hat{\tau}_i^1(\theta_i)\geq\lambda_i(\theta_i)\}} (\hat{\tau}_i^1(\theta_i)-\lambda_i(\theta_i))\, dF_i(\theta_i).$$

If it is shown that the value of the sup is less than or equal to the value of the inf, then the result is proved. By condition (ii), $\hat{\tau}_i^1(\theta_i)+\lambda_i(\theta_i)-2\theta_i$ is an increasing function over its domain. Since $\hat{\tau}_i^1(m_i)<\lambda_i(m_i)$, the desired inequality will hold if $\{\theta_i:\hat{\tau}_i^1(\theta_i)<\lambda_i(\theta_i)\}$ is an interval, which is implied by condition (ii). <u>QED</u>

When expected prices are higher in the signaling equilibrium than in the interim benchmark, each type of firm has lower expected profits in the signaling equilibrium. This is because the higher expected prices lower profits for any price charged by the firm and, except for the lowest cost type, the firm does not maximize first period expected profits. The interests of firms and society coincide in this case. The firms, however, cannot achieve the interim benchmark without some mechanism which implements it (since it is assumed that firms do not have the ability to independently verifiably disclose their costs, see Section 5).

Consider now the ex ante benchmark. The vector of first period equilibrium strategies for this benchmark is $\xi(\theta) = (C+\Delta(C))^{-1}(C\alpha+\Delta(C)\theta)$. Let Σ denote the variance-covariance matrix of θ, i.e., $\Sigma = E\{(\theta-E\theta)(\theta-E\theta)'\}$. (By the independence of θ_i and θ_j, Σ is a diagonal matrix.)

<u>Lemma 1</u>: Expected welfare under the ex ante benchmark is strictly lower than under the interim benchmark if, and only if,[17]

$$(10) \qquad tr[3C\Delta(C)^{-1}C-2C-\Delta(C)](C+\Delta(C))^{-1}C(C+\Delta(C))^{-1}\Delta(C)\Sigma > 0.$$

In particular, (10) is satisfied in the two firm case.

<u>Proof</u>: See the Appendix.

It seems plausible that (10) will generally hold because the welfare measure displays risk aversion with respect to each price, and prices in the ex ante benchmark are more variable with the same expected value than in the interim benchmark.[18] Unfortunately, it has not been possible to obtain general

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[17]The trace of the matrix A is denoted tr A.

[18]In the Appendix it is demonstrated that $c_{ii}[(C + \Delta(C))^{-1}]_{ii} > ½$. Also, ξ_i depends upon the full vector θ, not just θ_i.

conditions under which it can be shown that (10) either does or does not hold.[19] The two firm case suggests that from society's perspective, even if the ex ante benchmark were feasible, the interim benchmark is the most desirable.

5. Related Issues and Literature

The assumption that society has the means to force firms to share cost data at the end of the first period raises the possibility that firms may have the ability to independently and verifiably disclose their own costs. If this is technically feasible, then by the unravelling argument of Grossman [1981], all the firms would reveal their costs (the unravelling would start with the high cost firm, see Okuno-Fujiwara, Postlewaite and Suzumura [1986]). In this paper it is assumed that firms do not have the ability to verifiably disclose their costs. Given the incentives discussed in the Introduction, a claim by a firm that it is of high cost without independent proof or verification would not be believed by the other firms.

There is a recent strand of literature on the nonstrategic exchange of private information in oligopoly (Novshek and Sonnenschein [1982], Clarke [1983], Vives [1984], Gal-Or [1985, 1986a, 1986b], Katz [1985] and Shapiro [1986]). In particular the following question is asked: Does information sharing amongst oligopolists increase welfare and/or total profits? In the papers addressing this question, with some exceptions to be discussed,[20] the analysis is in the context of a one period model and in the absence of information sharing, the firms play a one-shot game of asymmetric information. The signaling equilibrium

[19] The source of difficulty is that the matrix $[3C\Delta(C)^{-1}C-2C-\Delta(C)]$ is not positive definite.

[20] Gal-Or [1986b] and Katz [1985].

of this paper captures the notion that in the absence of information sharing there are still important avenues by which firms can infer the private information of the other firms.

The literature on information sharing does not provide a ranking of the two benchmarks for the n firm case since the models studied there differ in certain important respects from the model studied here. Shapiro [1986] does study the homogeneous output Cournot oligopoly case with private information about costs and finds that the ex ante benchmark has higher expected welfare than the interim benchmark. This would appear to support the conjectured ranking (at least for the case of substitutes) since this type of result tends to reverse with changes in the strategic variable (see Vives [1984] and the next Section).

In general, games of simultaneous signaling have a major technical difficulty which makes existence of separating equilibria and subsequent analysis nontrivial (even given the standard single crossing property). If a strategy profile is a separating sequential equilibrium then it will, in general, have an unbounded derivative (see Theorem 1 and footnote 10). This implies that best response mappings will not be well behaved, and so existence problems cannot a priori be ruled out. While Mailath [1988] does prove existence under fairly general conditions, there is no uniqueness result under the same conditions. The linear structure of the model studied here has the property that a firm's expected payoff depends only on the <u>expected value</u> of the other firms' first period strategies, which is sufficient to give uniqueness. The first period strategies in the signaling equilibrium reflect the fundamental nonlinear nature of the first period strategies in signaling equilibria. In particular, the low cost firms price so as to maximize first period expected profits. This corresponds to the highest cost firms producing at the monopoly level in the first period in Milgrom and Roberts [1982] and the lowest productivity workers choosing no education in Spence [1973].

The papers of Gal-Or [1986b, 1988] and Katz [1985], which also study simultaneous signaling models, avoid the difficulty discussed above by specifying the games in such a way so as to obtain linear signaling equilibria. The papers by Gal-Or study duopolies and assume that the private information is drawn from the real line and has a particular distributional structure. The paper by Katz studies the two period analogue of Shapiro's [1986] model, employing a clever "perturbation" of that game which has linear signaling strategies and which approximates the original game.

6. Quantity Competition and General Demand

The choice of strategic variable is an awkward issue in industrial organization. In this paper, the assumption was made that firms compete through price since this does not involve an auctioneer. While Cournot competition has been justified as the subgame perfect equilibrium of a game in which firms first choose capacity and then compete through price by Kreps and Scheinkman [1982], this result holds only for their particular rationing rule (see Davidson and Deneckere [1986]). Furthermore, the model of Kreps and Scheinkman [1982] assumes complete information and so their results are not applicable to an incomplete information setting.

Nonetheless it is interesting to consider how the results of this paper would alter if firms competed through quantity rather than price. Nothing new is introduced technically. Firms now have an incentive for output expansion. Consider the case of substitutes (the goods can be perfect substitutes). A firm would like its competitors to reduce production. A competitor will reduce production if he believes the firm will produce a large quantity, which will occur if the firm has low costs. Thus, in the first period the firm has an incentive to increase production. A similar argument applies if the goods are complements. Note that this incentive to increase production is separate from the incentives discussed earlier in the literature which rely on entry deterrence. An interesting interpretation of the signaling equilibrium is that it provides an explanation of price wars which occur early in an industry's history.[21,22] Each firm expands output in an attempt to make his competitors believe its costs are low and that he thus should have a large market share. Observe that the welfare results will be reversed with quantity competition since quantities will now be larger rather than prices.

The game studied in this paper has several special features. The most restrictive feature is the assumption of linear demand. The general existence theorem in Mailath [1988] implies that the model (with some concavity assumptions on demand) will still have a separating equilibrium. There is however no expectation that there will be only one separating equilibrium path. Properties (i), (ii) and (iii) of Theorem 1 can still be shown to hold in any separating equilibrium. More general welfare statements however would be difficult to obtain.

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[21]I am grateful to John Roberts for suggesting this interpretation. The statement of the interpretation is taken from Milgrom and Roberts [1987].

[22]Parsons [1985], who studies a duopoly supergame in which one firm's unit cost is private information, also obtains a price war in the early stages of an industry.

Appendix

Fact: For $\hat{\theta}_i \geq \theta_i$, $\partial \Xi_i(p_i^1, \hat{\theta}_i, \theta_i)/\partial \hat{\theta}_i > 0$.

Proof: Differentiating Ξ_i with respect to $\hat{\theta}_i$ yields

$$\partial \Xi_i(p_i^1, \hat{\theta}_i, \theta_i)/\partial \hat{\theta}_i = 2\delta c_{ii} E(\xi_i^*(\hat{\theta}_i, \theta_i, \theta_{-i}) - \theta_i) \partial \xi_i^*(\hat{\theta}_i, \theta_i, \theta_{-i})/\partial \hat{\theta}_i.$$

Since $\xi_i^*(\hat{\theta}_i, \theta_i, \theta_{-i}) = \xi_i(\hat{\theta}_i, \theta_{-i}) - \frac{1}{2}(\hat{\theta}_i - \theta_i)$, $\partial \xi_i^*(\hat{\theta}_i, \theta_i, \theta_{-i})/\partial \hat{\theta}_i = \partial \xi_i(\hat{\theta}_i, \theta_{-i})/\partial \hat{\theta}_i - \frac{1}{2}$ $= c_{ii}[(C+\Delta(C))^{-1}]_{ii} - \frac{1}{2}$. Applying the formula for a partitioned inverse to $(C+\Delta(C))^{-1}$ gives

$$[(C + \Delta(C))^{-1}]_{11} = \left\{ 2c_{11} - [c_{12} \ldots c_{1n}] \begin{bmatrix} 2c_{22} & c_{23} \ldots & c_{2n} \\ \vdots & \vdots \quad \cdot & \vdots \\ c_{n2} & c_{n3} \ldots & 2c_{nn} \end{bmatrix}^{-1} \begin{bmatrix} c_{21} \\ \vdots \\ c_{n1} \end{bmatrix} \right\}^{-1}$$

$> (2c_{11})^{-1}$, since $C+\Delta(C)$ is symmetric and positive definite. Applying the appropriate permutation matrices and the above formula yields $\partial \xi_i(\theta)/\partial \theta_i > \frac{1}{2}$ for all i. Furthermore, $\xi_i^*(\hat{\theta}_i, \theta_i, \theta_{-i}) - \theta_i = \xi_i(\hat{\theta}_i, \theta_{-i}) - (\hat{\theta}_i + \theta_i)/2 \geq \xi_i(\hat{\theta}_i, \theta_{-i}) - \hat{\theta}_i > 0$. QED

Proposition: If (τ^1, τ^2) is a separating sequential equilibrium profile, then $\tau_i^1(m_i) = \phi_i(\tau_{-i}^1, m_i)$.

Proof: Suppose $p_i^1 = \phi_i(\tau_{-i}^1, m_i) \in \tau_i^1([m_i, M_i])$ and let $\hat{\theta}_i = (\tau_i^1)^{-1}(\phi_i(\tau_{-i}^1, m_i)) \neq m_i$. Then $\Xi_i(\tau_i^1(m_i), m_i, m_i) < \Xi_i(p_i^1, m_i, m_i) < \Xi_i(p_i^1, \hat{\theta}_i, \theta_i)$, and m_i profits by deviating. So suppose $\phi_i(\tau_{-i}^1, m_i) \notin \tau_i^1([m_i, M_i])$. Since the profile is sequential, a price of $\phi_i(\tau_{-i}^1, m_i)$ charged by firm i in the first period results in firms $j \neq i$ choosing optimal prices in the second period with respect to some belief, H_i, on $[m_i, M_i]$. If H_i puts all weight on one $\hat{\theta}_i \in [m_i, M_i]$, then the above sequence of inequalities (with the second weakly) applies and m_i profits by deviating. If H_i is not degenerate, firms $j \neq i$ play their strategies in the unique Bayesian equilibrium of the implied one-shot game, where i's unit cost is distributed according to H_i. Because of the linearity, these strategies are the same as in the equilibrium of the one-shot game where it is common knowledge that i's cost is equal to $\int\theta_i H_i(d\theta_i)$. Again the above sequence of inequalities apply. QED

Proof of Lemma 1: We need to show that (10) is equivalent to

$$\text{(A.1)} \qquad E\{(\xi(\theta)-\theta)'C(\xi(\theta)-\theta) - (\lambda(\theta)-\theta)'C(\lambda(\theta)-\theta)\} > 0.$$

Now, $\lambda(\theta) = E\xi(\theta)+(\theta-E\theta)/2$ and $\xi(\theta) = E\xi(\theta)+(C+\Delta(C))^{-1}\Delta(C)(\theta-E\theta)$. After substituting and canceling, the left hand side of (A.1) equals

$$\begin{aligned} &E\{-(\theta-E\theta)'C(\theta-E\theta)/4 + \theta'C(\theta-E\theta) - 2\theta'C(C + \Delta(C))^{-1}\Delta(C)(\theta-E\theta) \\ &\qquad + (\theta-E\theta)'\Delta(C)(C+\Delta(C))^{-1}C(C+\Delta(C))^{-1}\Delta(C)(\theta-E\theta)\} \\ &= (3/4)\mathrm{tr}C\Sigma - \mathrm{tr}(2C+\Delta(C))(C+\Delta(C))^{-1}C(C+\Delta(C))^{-1}\Delta(C)\Sigma, \end{aligned}$$

which after further simplification yields the left hand side of (10). Direct verification reveals that (10) holds for the two firm case. QED

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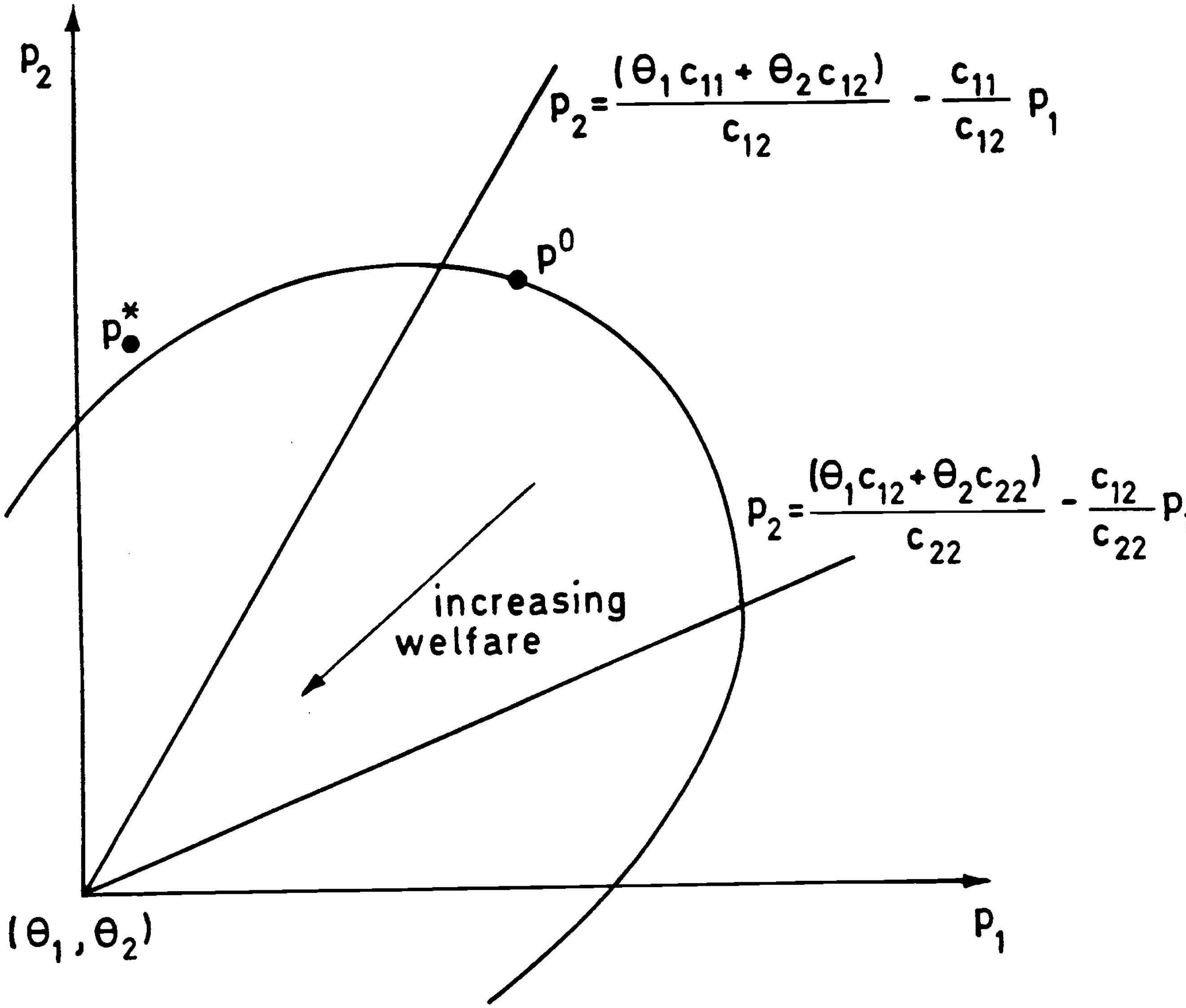

Figure 1: Isowelfare locus for the two firm, substitutes ($c_{12} < 0$) case.