

**For Referees Only, Not Intended for Publication.**

## B Appendix: Omitted Details of Some Derivations

### B.1 Derivation of Derivatives in (39):

For simplicity of notation, define:

$$\begin{aligned} A_1(z; \phi_G) &\equiv f \left[ \int_{\frac{f}{\phi_G}}^{\bar{\theta}_G} \int_{\underline{\theta}_P}^{\tau-f} h(\boldsymbol{\theta}) d\boldsymbol{\theta} \right], \\ B_1(z; \phi_G) &\equiv (p - c) \left[ \int_p^{\bar{\theta}_P} \int_{\underline{\theta}_G}^{\frac{\tau-p}{\phi_G^*}} h(\boldsymbol{\theta}) d\boldsymbol{\theta} \right], \\ C_1(z; \phi_G) &\equiv (\tau - c) \left[ \int_{\frac{\tau-p}{\phi_G}}^{\frac{f}{\phi_G}} \int_{\tau-\phi_G \theta_G}^{\bar{\theta}_P} h(\boldsymbol{\theta}) d\boldsymbol{\theta} + \int_{\frac{f}{\phi_G}}^{\bar{\theta}_G} \int_{\tau-f}^{\bar{\theta}_P} h(\boldsymbol{\theta}) d\boldsymbol{\theta} \right], \end{aligned}$$

so that

$$G_1(z; \phi_G) = A_1(z; \phi_G) + B_1(z; \phi_G) + C_1(z; \phi_G) - K\phi_G.$$

Differentiating with respect to  $f$ , we have:

$$\begin{aligned} \frac{\partial A_1(z)}{\partial f} &= \int_{\frac{f}{\pi}}^{\bar{\theta}_G} \int_{\underline{\theta}_P}^{\tau-f} h(\boldsymbol{\theta}) d\boldsymbol{\theta} - f \left[ \int_{\underline{\theta}_P}^{\tau-f} h\left(\frac{f}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P + \int_{\frac{f}{\pi}}^{\bar{\theta}_G} h(\theta_G, \tau-f) d\theta_G \right], \\ \frac{\partial B_1(z)}{\partial f} &= 0, \\ \frac{\partial C_1(z)}{\partial f} &= (\tau - c) \left[ \int_{\tau-f}^{\bar{\theta}_P} h\left(\frac{f}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P - \int_{\tau-f}^{\bar{\theta}_P} h\left(\frac{f}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P + \int_{\frac{f}{\pi}}^{\bar{\theta}_G} h(\theta_G, \tau-f) d\theta_G \right] \\ &= (\tau - c) \left[ \int_{\frac{f}{\pi}}^{\bar{\theta}_G} h(\theta_G, \tau-f) d\theta_G \right]. \end{aligned}$$

Thus,

$$\begin{aligned} \frac{\partial A_1(z^*)}{\partial f} &= \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} \int_{\underline{\theta}_P}^{p^*} h(\boldsymbol{\theta}) d\boldsymbol{\theta} - f^* \left[ \int_{\underline{\theta}_P}^{p^*} h\left(\frac{f^*}{\pi}, \theta_P\right) \frac{1}{\pi^*} d\theta_P + \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} h(\theta_G, p^*) d\theta_G \right], \\ \frac{\partial B_1(z^*)}{\partial f} &= 0, \\ \frac{\partial C_1(z^*)}{\partial f} &= (f^* + p^* - c) \left[ \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} h(\theta_G, p^*) d\theta_G \right]. \end{aligned}$$

So,

$$\begin{aligned}
\frac{\partial G_1(z^*)}{\partial f} &= \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} \left\{ \int_{\underline{\theta}_P}^{p^*} h(\boldsymbol{\theta}) d\theta_P + (p^* - c) h(\theta_G, p^*) \right\} d\theta_G - f^* \int_{\underline{\theta}_P}^{p^*} h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P \\
&= \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} \left\{ \int_{\underline{\theta}_P}^{p^*} \underbrace{\frac{h(\boldsymbol{\theta})}{\int_{\underline{\theta}_P}^{\bar{\theta}_P} h(\theta_G, \theta_P) d\theta_P} d\theta_P}_{h_P(\theta_P | \theta_G)} + (p^* - c) \underbrace{\frac{h(\theta_G, p^*)}{\int_{\underline{\theta}_P}^{\bar{\theta}_P} h(\theta_G, \theta_P) d\theta_P}}_{h_P(p^* | \theta_G)} \right\} \underbrace{\left( \int_{\underline{\theta}_P}^{\bar{\theta}_P} h(\theta_G, \theta_P) d\theta_P \right) d\theta_G}_{h_G(\theta_G)} d\theta_G \\
&\quad - f^* \int_{\underline{\theta}_P}^{p^*} \underbrace{\frac{h\left(\frac{f^*}{\pi^*}, \theta_P\right)}{\int_{\underline{\theta}_P}^{\bar{\theta}_P} h\left(\frac{f^*}{\pi^*}, \theta_P\right) d\theta_P} \frac{1}{\pi^*} d\theta_P}_{h_P(\theta_P | \frac{f^*}{\pi^*})} \underbrace{\int_{\underline{\theta}_P}^{\bar{\theta}_P} h\left(\frac{f^*}{\pi^*}, \theta_P\right) d\theta_P}_{h_G\left(\frac{f^*}{\pi^*}\right)} \\
&= \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} \left\{ \int_{\underline{\theta}_P}^{p^*} h_P(\theta_P | \theta_G) d\theta_P + (p^* - c) h_P(p^* | \theta_G) \right\} h_G(\theta_G) d\theta_G - \frac{f^*}{\pi^*} \int_{\underline{\theta}_P}^{p^*} h_P\left(\theta_P | \frac{f^*}{\pi^*}\right) d\theta_P h_G\left(\frac{f^*}{\pi^*}\right) \\
&= \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} \{H_P(p^* | \theta_G) + (p^* - c) h_P(p^* | \theta_G)\} h_G(\theta_G) d\theta_G - \frac{f^*}{\pi^*} H_P\left(p^* | \frac{f^*}{\pi^*}\right) h_G\left(\frac{f^*}{\pi^*}\right).
\end{aligned} \tag{B7}$$

Differentiating with respect to  $\tau$  yields:

$$\begin{aligned}
\frac{dA_1(z)}{d\tau} &= f \left[ \int_{\frac{f}{\pi}}^{\bar{\theta}_G} h(\theta_G, \tau - f) d\theta_G \right] \\
\frac{\partial B_1(z)}{\partial \tau} &= (p - c) \left[ \int_p^{\bar{\theta}_P} h\left(\frac{\tau - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P \right] \\
\frac{\partial C_1(z)}{\partial \tau} &= \int_{\frac{\tau-p}{\pi}}^{\frac{f}{\pi}} \int_{\tau - \pi\theta_G}^{\bar{\theta}_P} h(\boldsymbol{\theta}) d\boldsymbol{\theta} + \int_{\frac{f}{\pi}}^{\bar{\theta}_G} \int_{\tau-f}^{\bar{\theta}_P} h(\boldsymbol{\theta}) d\boldsymbol{\theta} \\
&\quad - (\tau - c) \left[ \int_p^{\bar{\theta}_P} h\left(\frac{\tau - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P + \int_{\frac{\tau-p}{\pi}}^{\frac{f}{\pi}} h(\theta_G, \tau - \pi\theta_G) d\theta_G \right] \\
&\quad - (\tau - c) \left[ \int_{\frac{f}{\pi}}^{\bar{\theta}_G} h(\theta_G, \tau - f) d\theta_G \right]
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{dA_1(z^*)}{d\tau} &= f^* \left[ \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} h(\theta_G, \tau - f^*) d\theta_G \right] \\
\frac{\partial B_1(z^*)}{\partial \tau} &= (p^* - c) \left[ \int_{p^*}^{\bar{\theta}_P} h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P \right] \\
\frac{\partial C_1(z^*)}{\partial \tau} &= \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} \int_{p^*}^{\bar{\theta}_P} h(\boldsymbol{\theta}) d\boldsymbol{\theta} - (p^* + f^* - c) \left\{ \left[ \int_{p^*}^{\bar{\theta}_P} h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P \right] + \left[ \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} h(\theta_G, p^*) d\theta_G \right] \right\}.
\end{aligned}$$

It follows that

$$\begin{aligned}
\frac{\partial G_1(z^*)}{\partial \tau} &= \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} \int_{p^*}^{\bar{\theta}_P} h(\boldsymbol{\theta}) d\boldsymbol{\theta} - f^* \left[ \int_{p^*}^{\bar{\theta}_P} h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P \right] - (p^* - c) \left[ \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} h(\theta_G, p^*) d\boldsymbol{\theta} \right] \\
&= \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} \left\{ \int_{p^*}^{\bar{\theta}_P} h(\boldsymbol{\theta}) d\theta_P - (p^* - c) h(\theta_G, p^*) \right\} d\theta_G - f^* \left[ \int_{p^*}^{\bar{\theta}_P} h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P \right] \\
/\text{same steps as in (B7)}/ &= \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} \left\{ [1 - H_P(p^* | \theta_G)] - (p^* - c) h_P(p^* | \theta_G) \right\} h_G(\theta_G) d\theta_G - \frac{f^*}{\pi^*} \left[ 1 - H_P\left(p^* | \frac{f^*}{\pi^*}\right) \right] h_G\left(\frac{f^*}{\pi^*}\right).
\end{aligned}$$

Differentiating with respect to  $p$  yields:

$$\begin{aligned}
\frac{\partial A_1(z)}{\partial p} &= 0, \\
\frac{\partial B_1(z)}{\partial p} &= \int_p^{\bar{\theta}_P} \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} h(\boldsymbol{\theta}) d\boldsymbol{\theta} - (p - c) \left[ \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} h(\theta_G, p) d\theta_G + \int_p^{\bar{\theta}_P} h\left(\frac{\tau-p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P \right], \\
\frac{\partial C_1(z)}{\partial p} &= (\tau - c) \left[ \int_p^{\bar{\theta}_P} h\left(\frac{\tau-p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P \right].
\end{aligned}$$

Hence,

$$\begin{aligned}
\frac{\partial A_1(z^*)}{\partial p} &= 0, \\
\frac{\partial B_1(z^*)}{\partial p} &= \int_{p^*}^{\bar{\theta}_P} \int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} h(\boldsymbol{\theta}) d\boldsymbol{\theta} - (p^* - c) \left[ \int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} h(\theta_G, p^*) d\theta_G + \int_{p^*}^{\bar{\theta}_P} h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P \right], \\
\frac{\partial C_1(z^*)}{\partial p} &= (f^* + p^* - c) \left[ \int_{p^*}^{\bar{\theta}_P} h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P \right],
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial G_1(z^*)}{\partial p} &= \int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} \left[ \int_{p^*}^{\bar{\theta}_P} h(\boldsymbol{\theta}) d\theta_P - (p^* - c) h(\theta_G, p^*) \right] d\theta_G + f^* \left[ \int_{p^*}^{\bar{\theta}_P} h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P \right] \\
/\text{same steps as in (B7)}/ &= \int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} \left[ [(1 - H_P(p^* | \theta_G)) - (p^* - c) h_P(p^* | \theta_G)] \right] h_G(\theta_G) d\theta_G \\
&\quad + \frac{f^*}{\pi^*} \left[ 1 - H_P\left(p^* | \frac{f^*}{\pi^*}\right) \right] h_G\left(\frac{f^*}{\pi^*}\right).
\end{aligned}$$

## B.2 Derivation of Derivatives in (40):

Let

$$\begin{aligned}
D_1(z) &= \int_{\frac{f}{\pi}}^{\bar{\theta}_G} \int_{\underline{\theta}_P}^{\tau-f} \pi \theta_G h(\boldsymbol{\theta}) d\boldsymbol{\theta}, \\
E_1(z) &= \int_p^{\bar{\theta}_P} \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} (\theta_P - c) h(\boldsymbol{\theta}) d\boldsymbol{\theta}, \\
H_1(z) &= \int_{\frac{\tau-p}{\pi}}^{\frac{f}{\pi}} \int_{\tau-\pi\theta_G}^{\bar{\theta}_P} (\pi \theta_G + \theta_P - c) h(\boldsymbol{\theta}) d\boldsymbol{\theta} + \int_{\frac{f}{\pi}}^{\bar{\theta}_G} \int_{\tau-f}^{\bar{\theta}_P} (\pi \theta_G + \theta_P - c) h(\boldsymbol{\theta}) d\boldsymbol{\theta}.
\end{aligned}$$

Hence,

$$\begin{aligned}\frac{\partial D_1(z)}{\partial f} &= - \int_{\underline{\theta}_P}^{\tau-f} f h\left(\frac{f}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P - \int_{\frac{f}{\pi}}^{\bar{\theta}_G} \pi \theta_G h(\theta_G, \tau-f) d\theta_G \\ \frac{\partial D_1(z)}{\partial p} &= 0 \\ \frac{\partial D_1(z)}{\partial \tau} &= \int_{\frac{f}{\pi}}^{\bar{\theta}_G} \pi \theta_G h(\theta_G, \tau-f) d\theta_G\end{aligned}$$

and thus,

$$\begin{aligned}\frac{\partial D_1(z^*)}{\partial f} &= - \int_{\underline{\theta}_P}^{p^*} f^* h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P - \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} \pi^* \theta_G h(\theta_G, p^*) d\theta_G \\ \frac{\partial D_1(z^*)}{\partial p} &= 0 \\ \frac{\partial D_1(z^*)}{\partial \tau} &= \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} \pi^* \theta_G h(\theta_G, p^*) d\theta_G.\end{aligned}\tag{B8}$$

Similarly,

$$\begin{aligned}\frac{\partial E_1(z)}{\partial f} &= 0, \\ \frac{\partial E_1(z)}{\partial p} &= - \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} (p-c) h(\theta_G, p) d\theta_G - \int_p^{\bar{\theta}_P} (\theta_P - c) h\left(\frac{\tau-p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P, \\ \frac{\partial E_1(z)}{\partial \tau} &= \int_p^{\bar{\theta}_P} (\theta_P - c) h\left(\frac{\tau-p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P,\end{aligned}$$

thus,

$$\begin{aligned}\frac{\partial E_1(z^*)}{\partial f} &= 0 \\ \frac{\partial E_1(z^*)}{\partial p} &= - \int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} (p^* - c) h(\theta_G, p^*) d\theta_G - \int_{p^*}^{\bar{\theta}_P} (\theta_P - c) h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P \\ \frac{\partial E_1(z^*)}{\partial \tau} &= \int_{p^*}^{\bar{\theta}_P} (\theta_P - c) h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P.\end{aligned}\tag{B9}$$

Finally,

$$\begin{aligned}
\frac{\partial H_1(z)}{\partial f} &= \int_{\tau-f}^{\bar{\theta}_P} (f + \theta_P - c) h\left(\frac{f}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P - \int_{\tau-f}^{\bar{\theta}_P} (f + \theta_P - c) h\left(\frac{f}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P \\
&\quad + \int_{\frac{f}{\pi}}^{\bar{\theta}_G} (\pi\theta_G + \tau - f - c) h(\theta_G, \tau - f) d\theta_G \\
&= \int_{\frac{f}{\pi}}^{\bar{\theta}_G} (\pi\theta_G + \tau - f - c) h(\theta_G, \tau - f) \\
\frac{\partial H_1(z)}{\partial p} &= \int_p^{\bar{\theta}_P} (\tau - p + \theta_P - c) h\left(\frac{\tau - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P \\
\frac{\partial H_1(z)}{\partial \tau} &= - \int_p^{\bar{\theta}_P} (\tau - p + \theta_P - c) h\left(\frac{\tau - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P - \int_{\frac{\tau-p}{\pi}}^{\frac{f}{\pi}} (\tau - c) h(\theta_G, \tau - \pi\theta_G) d\theta_G \\
&\quad - \int_{\frac{f}{\pi}}^{\bar{\theta}_G} (\pi\theta_G + \tau - f - c) h(\theta_G, \tau - f) d\theta_G
\end{aligned}$$

thus,

$$\begin{aligned}
\frac{\partial H_1(z^*)}{\partial f} &= \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} (\pi^*\theta_G + p^* - c) h(\theta_G, p^*) d\theta_G \\
\frac{\partial H_1(z^*)}{\partial p} &= \int_{p^*}^{\bar{\theta}_P} (f^* + \theta_P - c) h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P \\
\frac{\partial H_1(z^*)}{\partial \tau} &= - \int_{p^*}^{\bar{\theta}_P} (f^* + \theta_P - c) h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P - \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} (\pi^*\theta_G + p^* - c) h(\theta_G, p^*) d\theta_G.
\end{aligned} \tag{B10}$$

Hence,

$$\begin{aligned}
\frac{\partial S_1(z^*)}{\partial f} &= \frac{\partial D_1(z^*)}{\partial f} + \frac{\partial E_1(z^*)}{\partial f} + \frac{\partial H_1(z^*)}{\partial f} \\
&= - \int_{\underline{\theta}_P}^{p^*} f^* h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P - \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} \pi^*\theta_G h(\theta_G, p^*) d\theta_G + \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} (\pi^*\theta_G + p^* - c) h(\theta_G, p^*) d\theta_G \\
&= - \int_{\underline{\theta}_P}^{p^*} \frac{f^*}{\pi^*} h\left(\frac{f^*}{\pi^*}, \theta_P\right) d\theta_P + \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} (p^* - c) h(\theta_G, p^*) d\theta_G \\
&= \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} (p^* - c) h_P(p^* | \theta_G) h_G(\theta_G) d\theta_G - \frac{f^*}{\pi^*} H_P\left(p^* | \frac{f^*}{\pi^*}\right) h_G\left(\frac{f^*}{\pi^*}\right)
\end{aligned}$$

and,

$$\begin{aligned}
\frac{\partial S_1(z^*)}{\partial p} &= \frac{\partial D_1(z^*)}{\partial p} + \frac{\partial E_1(z^*)}{\partial p} + \frac{\partial H_1(z^*)}{\partial p} \\
&= - \int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} (p^* - c) h(\theta_G, p^*) d\theta_G - \int_{p^*}^{\bar{\theta}_P} (\theta_P - c) h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P \\
&\quad + \int_{p^*}^{\bar{\theta}_P} (f^* + \theta_P - c) h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P \\
&= - \int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} (p^* - c) h(\theta_G, p^*) d\theta_G + \int_{p^*}^{\bar{\theta}_P} \frac{f^*}{\pi^*} h\left(\frac{f^*}{\pi^*}, \theta_P\right) d\theta_P \\
&= - \int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} (p^* - c) h(\theta_G, p^*) d\theta_G + \frac{f^*}{\pi^*} \left[1 - H_P\left(p^* | \frac{f^*}{\pi^*}\right)\right] h_G\left(\frac{f^*}{\pi^*}\right),
\end{aligned}$$

and,

$$\begin{aligned}
\frac{\partial S_1(z^*)}{\partial \tau} &= \frac{\partial D_1(z^*)}{\partial \tau} + \frac{\partial E_1(z^*)}{\partial \tau} + \frac{\partial H_1(z^*)}{\partial \tau} \\
&= \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} \pi^* \theta_G h(\theta_G, p^*) d\theta + \int_{p^*}^{\bar{\theta}_P} (\theta_P - c) h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P \\
&\quad - \int_{p^*}^{\bar{\theta}_P} (f^* + \theta_P - c) h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P - \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} (\pi^* \theta_G + p^* - c) h(\theta_G, p^*) d\theta_G \\
&= - \int_{p^*}^{\bar{\theta}_P} \frac{f^*}{\pi^*} h\left(\frac{f^*}{\pi^*}, \theta_P\right) d\theta_P - \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} (p^* - c) h(\theta_G, p^*) d\theta_G \\
&= - \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} (p^* - c) h_P(p^* | \theta_G) h_G(\theta_G) d\theta_G - \frac{f^*}{\pi^*} \left[ 1 - H_P\left(p^* \middle| \frac{f^*}{\pi^*}\right) \right] h_G\left(\frac{f^*}{\pi^*}\right).
\end{aligned}$$

### B.3 Calculations in Proving Lemma 3

- **Part 1:**  $DG_1(z^*) = DG_2(z^*)$ :

Write  $G_2(z) = A_2(z) + B_2(z) + C_2(z) - K\pi$  where:

$$\begin{aligned}
A_2(z) &\equiv f \left[ \int_{\frac{f}{\pi}}^{\frac{\tau-p}{\pi}} \int_{\underline{\theta}_P}^{\pi\theta_G+p-f} h(\theta) d\theta + \int_{\frac{\tau-p}{\pi}}^{\bar{\theta}_G} \int_{\underline{\theta}_P}^{\tau-f} h(\theta) d\theta \right] \\
B_2(z) &\equiv (p - c) \left[ \int_p^{\tau-f} \int_{\underline{\theta}_G}^{\frac{\theta_P+f-p}{\pi}} h(\theta) d\theta + \int_{\tau-f}^{\bar{\theta}_P} \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} h(\theta) d\theta \right] \\
C_2(z) &\equiv (\tau - c) \left[ \int_{\frac{\tau-p}{\pi}}^{\bar{\theta}_G} \int_{\tau-f}^{\bar{\theta}_P} h(\theta) d\theta \right].
\end{aligned}$$

Differentiating  $A_2$  we get:

$$\begin{aligned}
\frac{\partial A_2(z)}{\partial f} &= \left[ \int_{\frac{f}{\pi}}^{\frac{\tau-p}{\pi}} \int_{\underline{\theta}_P}^{\pi\theta_G+p-f} h(\theta) d\theta + \int_{\frac{\tau-p}{\pi}}^{\bar{\theta}_G} \int_{\underline{\theta}_P}^{\tau-f} h(\theta) d\theta \right] \\
&\quad - f \left[ \int_{\underline{\theta}_P}^p h\left(\frac{f}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P + \int_{\frac{f}{\pi}}^{\frac{\tau-p}{\pi}} h(\theta_G, \pi\theta_G + p - f) d\theta_G + \int_{\frac{\tau-p}{\pi}}^{\bar{\theta}_G} h(\theta_G, \tau - f) d\theta_G \right] \\
\frac{\partial A_2(z)}{\partial p} &= -f \int_{\underline{\theta}_P}^{\tau-f} h\left(\frac{\tau-p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P + f \int_{\frac{f}{\pi}}^{\frac{\tau-p}{\pi}} h(\theta_G, \pi\theta_G + p - f) d\theta_G + f \int_{\underline{\theta}_P}^{\tau-f} h\left(\frac{\tau-p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P \\
&= f \int_{\frac{f}{\pi}}^{\frac{\tau-p}{\pi}} h(\theta_G, \pi\theta_G + p - f) d\theta_G \\
\frac{\partial A_2(z)}{\partial \tau} &= f \int_{\underline{\theta}_P}^{\tau-f} h\left(\frac{\tau-p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P - f \int_{\underline{\theta}_P}^{\tau-f} h\left(\frac{\tau-p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P + f \int_{\frac{\tau-p}{\pi}}^{\bar{\theta}_G} h(\theta_G, \tau - f) d\theta_G \\
&= f \int_{\frac{\tau-p}{\pi}}^{\bar{\theta}_G} h(\theta_G, \tau - f) d\theta_G
\end{aligned}$$

Hence,

$$\begin{aligned}\frac{\partial A_2(z^*; \pi)}{\partial f} &= \int_{\frac{f^*}{\pi}}^{\bar{\theta}_G} \int_{\underline{\theta}_P}^{p^*} h(\boldsymbol{\theta}) d\boldsymbol{\theta} - f^* \left[ \int_{\underline{\theta}_P}^{p^*} h\left(\frac{f^*}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P + \int_{\frac{f^*}{\pi}}^{\bar{\theta}_G} h(\theta_G, p^*) d\theta_G \right] \\ \frac{\partial A_2(z^*; \pi)}{\partial p} &= 0 \\ \frac{\partial A_2(z^*; \pi)}{\partial \tau} &= f^* \int_{\frac{f^*}{\pi}}^{\bar{\theta}_G} h(\theta_G, p^*) d\theta_G.\end{aligned}$$

Similarly,

$$\begin{aligned}\frac{\partial B_2(z)}{\partial f} &= (p - c) \left[ - \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} h(\theta_G, \tau - f) d\theta_G + \int_p^{\tau-f} h\left(\frac{\theta_P + f - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P + \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} h(\theta_G, \tau - f) d\theta_G \right] \\ &= (p - c) \left[ \int_p^{\tau-f} h\left(\frac{\theta_P + f - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P \right] \\ \frac{\partial B_2(z)}{\partial p} &= \int_p^{\tau-f} \int_{\underline{\theta}_G}^{\frac{\theta_P + f - p}{\pi}} h(\boldsymbol{\theta}) d\boldsymbol{\theta} + \int_{\tau-f}^{\bar{\theta}_P} \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} h(\boldsymbol{\theta}) d\boldsymbol{\theta} \\ &\quad - (p - c) \left[ \int_{\underline{\theta}_G}^{\frac{f}{\pi}} h(\theta_G, p) d\theta_G + \int_p^{\tau-f} h\left(\frac{\theta_P + f - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P + \int_{\tau-f}^{\bar{\theta}_P} h\left(\frac{\tau-p}{\pi}, \theta_P\right) d\theta_P \right] \\ \frac{\partial B_2(z)}{\partial \tau} &= (p - c) \left[ \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} h(\theta_G, \tau - f) d\theta_G - \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} h(\theta_G, \tau - f) d\theta_G + \int_{\tau-f}^{\bar{\theta}_P} h\left(\frac{\tau-p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_G \right] \\ &= (p - c) \left[ \int_{\tau-f}^{\bar{\theta}_P} h\left(\frac{\tau-p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_G \right].\end{aligned}$$

Hence,

$$\begin{aligned}\frac{\partial B_2(z^*)}{\partial f} &= 0 \\ \frac{\partial B_2(z^*)}{\partial p} &= \int_{p^*}^{\bar{\theta}_P} \int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} h(\boldsymbol{\theta}) d\boldsymbol{\theta} - (p^* - c) \left[ \int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} h(\theta_G, p^*) d\theta_G + \int_{p^*}^{\bar{\theta}_P} h\left(\frac{f^*}{\pi^*}, \theta_P\right) d\theta_P \right] \\ \frac{\partial B_2(z^*)}{\partial \tau} &= (p^* - c) \left[ \int_{p^*}^{\bar{\theta}_P} h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_G \right].\end{aligned}$$

Similarly,

$$\begin{aligned}\frac{\partial C_2(z)}{\partial f} &= (\tau - c) \left[ \int_{\frac{\tau-p}{\pi}}^{\bar{\theta}_G} h(\theta_G, \tau - f) d\theta_G \right] \\ \frac{\partial C_2(z)}{\partial p} &= (\tau - c) \left[ \int_{\tau-f}^{\bar{\theta}_P} h\left(\frac{\tau-p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P \right] \\ \frac{\partial C_2(z)}{\partial \tau} &= \int_{\frac{\tau-p}{\pi}}^{\bar{\theta}_G} \int_{\tau-f}^{\bar{\theta}_P} h(\boldsymbol{\theta}) d\boldsymbol{\theta} - (\tau - c) \left[ \int_{\tau-f}^{\bar{\theta}_P} h\left(\frac{\tau-p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P + \int_{\frac{\tau-p}{\pi}}^{\bar{\theta}_G} h(\theta_G, \tau - f) d\theta_G \right].\end{aligned}$$

Hence,

$$\begin{aligned}\frac{\partial C_2(z^*)}{\partial f} &= (f^* + p^* - c) \left[ \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} h(\theta_G, p^*) d\theta_G \right] \\ \frac{\partial C_2(z^*)}{\partial p} &= (f^* + p^* - c) \left[ \int_{p^*}^{\bar{\theta}_P} h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P \right] \\ \frac{\partial C_2(z^*)}{\partial \tau} &= \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} \int_{p^*}^{\bar{\theta}_P} h(\boldsymbol{\theta}) d\boldsymbol{\theta} - (f^* + p^* - c) \left[ \int_{p^*}^{\bar{\theta}_P} h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P + \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} h(\theta_G, p^*) d\theta_G \right]\end{aligned}$$

Combining terms we get that

$$\begin{aligned}\frac{\partial G_2(z^*)}{\partial f} &= \frac{\partial A_2(z^*)}{\partial f} + \frac{\partial B_2(z^*)}{\partial f} + \frac{\partial C_2(z^*)}{\partial f} \\ &= \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} \int_{\underline{\theta}_P}^{p^*} h(\boldsymbol{\theta}) d\boldsymbol{\theta} - f^* \left[ \int_{\underline{\theta}_P}^{p^*} h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P + \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} h(\theta_G, p^*) d\theta_G \right] \\ &\quad + (f^* + p^* - c) \left[ \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} h(\theta_G, p^*) d\theta_G \right] \\ &= \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} \left\{ \int_{\underline{\theta}_P}^{p^*} h(\boldsymbol{\theta}) d\theta_P + (p^* - c) h(\theta_G, p^*) \right\} d\theta_G - f^* \int_{\underline{\theta}_P}^{p^*} h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P \\ &= \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} \left\{ \int_{\underline{\theta}_P}^{p^*} h_P(\theta_P | \theta_G) d\theta_P + (p^* - c) h_P(p^* | \theta_G) \right\} h_G(\theta_G) d\theta_G - \frac{f^*}{\pi^*} \int_{\underline{\theta}_P}^{p^*} h_P\left(\theta_P | \frac{f^*}{\pi^*}\right) d\theta_P h_G\left(\frac{f^*}{\pi^*}\right) \\ &= \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} \{H_P(p^* | \theta_G) + (p^* - c) h_P(p^* | \theta_G)\} h_G(\theta_G) d\theta_G - \frac{f^*}{\pi^*} H_P\left(p^* | \frac{f^*}{\pi^*}\right) h_G\left(\frac{f^*}{\pi^*}\right) \\ &= \frac{\partial G_1(z^*)}{\partial f},\end{aligned}$$

and,

$$\begin{aligned}\frac{\partial G_2(z^*)}{\partial p} &= \frac{\partial A_2(z^*)}{\partial p} + \frac{\partial B_2(z^*)}{\partial p} + \frac{\partial C_2(z^*)}{\partial p} \\ &= \int_{p^*}^{\bar{\theta}_P} \int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} h(\boldsymbol{\theta}) d\boldsymbol{\theta} - (p^* - c) \left[ \int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} h(\theta_G, p^*) d\theta_G + \int_{p^*}^{\bar{\theta}_P} h\left(\frac{f^*}{\pi^*}, \theta_P\right) d\theta_P \right] \\ &\quad + (f^* + p^* - c) \left[ \int_{p^*}^{\bar{\theta}_P} h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P \right] \\ &= \int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} \left\{ \int_{p^*}^{\bar{\theta}_P} h(\boldsymbol{\theta}) d\theta_P - (p^* - c) h(\theta_G, p^*) \right\} d\theta_G + f^* \left[ \int_{p^*}^{\bar{\theta}_P} h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P \right] \\ &= \int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} \{1 - H_P(p^* | \theta_G) - (p^* - c) h_P(p^* | \theta_G)\} h_G(\theta_G) d\theta_G + f^* \left[ 1 - H_P\left(p^* | \frac{f^*}{\pi^*}\right) \right] h_G\left(\frac{f^*}{\pi^*}\right) \\ &= \frac{\partial G_1(z^*)}{\partial p}\end{aligned}$$

$$\begin{aligned}
\frac{\partial G_2(z^*)}{\partial \tau} &= \frac{\partial A_2(z^*)}{\partial \tau} + \frac{\partial B_2(z^*)}{\partial \tau} + \frac{\partial C_2(z^*)}{\partial \tau} \\
&= f^* \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} h(\theta_G, p^*) d\theta_G + (p^* - c) \left[ \int_{p^*}^{\bar{\theta}_P} h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P \right] \\
&\quad + \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} \int_{p^*}^{\bar{\theta}_P} h(\boldsymbol{\theta}) d\boldsymbol{\theta} - (f^* + p^* - c) \left[ \int_{p^*}^{\bar{\theta}_P} h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P + \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} h(\theta_G, p^*) d\theta_G \right] \\
&= \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} \left\{ \int_{p^*}^{\bar{\theta}_P} h(\boldsymbol{\theta}) d\theta_P - (p^* - c) h(\theta_G, p^*) \right\} d\theta_G - f^* \left[ \int_{p^*}^{\bar{\theta}_P} h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P \right] \\
&= \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} \{1 - H_P(p^* | \theta_G) - (p^* - c) h_P(p^* | \theta_G)\} h_G(\theta_G) d\theta_G - \frac{f^*}{\pi^*} \left[ 1 - H_P\left(p^* | \frac{f^*}{\pi^*}\right) h_G\left(\frac{f^*}{\pi^*}\right) \right] \\
&= \frac{\partial G_1(z^*)}{\partial \tau}
\end{aligned}$$

• **Part 2:**  $DS_1(z^*) = DS_2(z^*)$ :

Write  $S_2(z) = D_2(z) + E_2(z) + F_2(z) - K\pi$  where

$$\begin{aligned}
D_2(z) &= \int_{\frac{f}{\pi}}^{\frac{\tau-p}{\pi}} \int_{\underline{\theta}_P}^{\pi\theta_G+p-f} \pi\theta_G h(\boldsymbol{\theta}) d\boldsymbol{\theta} + \int_{\frac{\tau-p}{\pi}}^{\bar{\theta}_G} \int_{\underline{\theta}_P}^{\tau-f} \pi\theta_G h(\boldsymbol{\theta}) d\boldsymbol{\theta} \\
E_2(z) &= \int_p^{\tau-f} \int_{\underline{\theta}_G}^{\frac{\theta_P+f-p}{\pi}} (\theta_P - c) h(\boldsymbol{\theta}) d\boldsymbol{\theta} + \int_{\tau-f}^{\bar{\theta}_P} \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} (\theta_P - c) h(\boldsymbol{\theta}) d\boldsymbol{\theta} \\
F_2(z) &= \int_{\frac{\tau-p}{\pi}}^{\bar{\theta}_G} \int_{\tau-f}^{\bar{\theta}_P} (\pi\theta_G + \theta_P - c) h(\boldsymbol{\theta}) d\boldsymbol{\theta}.
\end{aligned}$$

Thus,

$$\begin{aligned}
\frac{\partial D_2(z)}{\partial f} &= - \int_{\underline{\theta}_P}^p \frac{f}{\pi} h\left(\frac{f}{\pi}, \theta_P\right) d\theta_P - \int_{\frac{f}{\pi}}^{\frac{\tau-p}{\pi}} \pi\theta_G h(\theta_G, \pi\theta_G + p - f) d\theta_G - \int_{\frac{\tau-p}{\pi}}^{\bar{\theta}_G} \pi\theta_G h(\theta_G, \tau - f) d\theta_G \\
\frac{\partial D_2(z)}{\partial p} &= - \int_{\underline{\theta}_P}^{\tau-f} \frac{\tau-p}{\pi} h\left(\frac{\tau-p}{\pi}, \theta_P\right) d\theta_P + \int_{\frac{f}{\pi}}^{\frac{\tau-p}{\pi}} \pi\theta_G h(\theta_G, \pi\theta_G + p - f) d\theta_P + \int_{\underline{\theta}_P}^{\tau-f} \frac{\tau-p}{\pi} h\left(\frac{\tau-p}{\pi}, \theta_P\right) d\theta_P \\
&= \int_{\frac{f}{\pi}}^{\frac{\tau-p}{\pi}} \pi\theta_G h(\theta_G, \pi\theta_G + p - f) d\theta_P \\
\frac{\partial D_2(z)}{\partial \tau} &= \int_{\underline{\theta}_P}^{\tau-f} \frac{\tau-p}{\pi} h\left(\frac{\tau-p}{\pi}, \theta_P\right) d\theta_P - \int_{\underline{\theta}_P}^{\tau-f} \frac{\tau-p}{\pi} h\left(\frac{\tau-p}{\pi}, \theta_P\right) d\theta_P + \int_{\frac{\tau-p}{\pi}}^{\bar{\theta}_G} \pi\theta_G h(\theta_G, \tau - f) d\theta_G \\
&= \int_{\frac{\tau-p}{\pi}}^{\bar{\theta}_G} \pi\theta_G h(\theta_G, \tau - f) d\theta_G,
\end{aligned}$$

so,

$$\begin{aligned}
\frac{\partial D_2(z^*)}{\partial f} &= - \int_{\underline{\theta}_P}^{p^*} \frac{f^*}{\pi^*} h\left(\frac{f^*}{\pi^*}, \theta_P\right) d\theta_P - \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} \pi^* \theta_G h(\theta_G, p^*) d\theta_G \\
\frac{\partial D_2(z^*)}{\partial p} &= 0 \\
\frac{\partial D_2(z^*)}{\partial \tau} &= \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} \pi^* \theta_G h(\theta_G, p^*) d\theta_G,
\end{aligned}$$

which is the same as in (B8).

Similarly,

$$\begin{aligned}
\frac{\partial E_2(z)}{\partial f} &= - \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} (\tau - f - c) h(\theta_G, \tau - f) d\theta_G + \int_p^{\tau-f} (\theta_P - c) h\left(\frac{\theta_P + f - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P \\
&\quad + \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} (\tau - f - c) h(\theta_G, \tau - f) d\theta_G \\
&= \int_p^{\tau-f} (\theta_P - c) h\left(\frac{\theta_P + f - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P \\
\frac{\partial E_2(z)}{\partial p} &= - \int_{\underline{\theta}_G}^{\frac{f}{\pi}} (p - c) h(\theta_G, p) d\theta_G - \int_p^{\tau-f} (\theta_P - c) h\left(\frac{\theta_P + f - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P \\
&\quad - \int_{\tau-f}^{\bar{\theta}_P} (\theta_P - c) h\left(\frac{\tau - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P \\
\frac{\partial E_2(z)}{\partial \tau} &= \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} (\tau - f - c) h(\theta_G, \tau - f) d\theta_G - \int_{\underline{\theta}_G}^{\frac{\tau-p}{\pi}} (\tau - f - c) h(\theta_G, \tau - f) d\theta_G \\
&\quad + \int_{\tau-f}^{\bar{\theta}_P} (\theta_P - c) h\left(\frac{\tau - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P \\
&= \int_{\tau-f}^{\bar{\theta}_P} (\theta_P - c) h\left(\frac{\tau - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P,
\end{aligned}$$

so,

$$\begin{aligned}
\frac{\partial E_2(z^*)}{\partial f} &= 0 \\
\frac{\partial E_2(z^*)}{\partial p} &= - \int_{\underline{\theta}_G}^{\frac{f^*}{\pi^*}} (p^* - c) h(\theta_G, p^*) d\theta_G - \int_{p^*}^{\bar{\theta}_P} (\theta_P - c) h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P \\
\frac{\partial E_2(z^*)}{\partial \tau} &= \int_{p^*}^{\bar{\theta}_P} (\theta_P - c) h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P,
\end{aligned}$$

which is the same as the expressions in (B9).

Finally,

$$\begin{aligned}
\frac{\partial F_2(z)}{\partial f} &= \int_{\frac{\tau-p}{\pi^*}}^{\bar{\theta}_G} (\pi\theta_G + \tau - f - c) h(\theta_G, \tau - f) d\theta_G \\
\frac{\partial F_2(z)}{\partial p} &= \int_{\tau-f}^{\bar{\theta}_P} (\tau - p + \theta_P - c) h\left(\frac{\tau - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P \\
\frac{\partial F_2(z)}{\partial \tau} &= - \int_{\tau-f}^{\bar{\theta}_P} (\tau - p + \theta_P - c) h\left(\frac{\tau - p}{\pi}, \theta_P\right) \frac{1}{\pi} d\theta_P - \int_{\frac{\tau-p}{\pi}}^{\bar{\theta}_G} (\pi\theta_G + \tau - f - c) h(\theta_G, \tau - f) d\theta_G,
\end{aligned}$$

so,

$$\begin{aligned}
\frac{\partial F_2(z^*)}{\partial f} &= \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} (\pi^*\theta_G + p^* - c) h(\theta_G, p^*) d\theta_G \\
\frac{\partial F_2(z^*)}{\partial p} &= \int_{p^*}^{\bar{\theta}_P} (f^* + \theta_P - c) h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P \\
\frac{\partial F_2(z^*)}{\partial \tau} &= - \int_{p^*}^{\bar{\theta}_P} (f^* + \theta_P - c) h\left(\frac{f^*}{\pi^*}, \theta_P\right) \frac{1}{\pi^*} d\theta_P - \int_{\frac{f^*}{\pi^*}}^{\bar{\theta}_G} [\pi^*\theta_G + p^* - c] h(\theta_G, p^*) d\theta_G
\end{aligned}$$

which is the same as the expressions in (B10). Since all the components are identical the result follows.