Excitation of multiple surface-plasmon-polariton waves and waveguide modes in a 1D photonic crystal atop a 2D metal grating

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ABSTRACT

The Floquet theory and the transfer-matrix approach were used to investigate the excitation of surface-plasmon-polariton (SPP) waves and waveguide modes in a structure comprising a one-dimensional photonic crystal (1D PC) of finite thickness on top of a planar thick metallic layer. The solutions of the relevant dispersion equations were used to predict the excitation of multiple SPP waves and waveguide modes when the metallic layer is patterned as a two-dimensional (2D) surface-relief grating. The same structure was experimentally fabricated and optically characterized to validate the theoretical approach. Both the theoretical and experimental results show broadband coupling of incident light of either linear polarization state over a broad range of the angle of incidence. This structure has potential applications in planar optical concentrators.

Keywords: SPP wave, waveguide mode, Floquet theorem, grating coupling, planar optical concentrator

1. INTRODUCTION

Surface plasmon-polariton (SPP) waves are electromagnetic surface waves guided by the planar interface between a dielectric material and a conductor, evanescently confined in the perpendicular direction.[1] SPP waves are commonly exploited for sensing,[2] biosensing,[3] optical filtering,[4] and photodetection.[5] Of particular interest in recent years has been their applications for photovoltaics,[6] and photoelectrochemical cells[7] because SPP waves can potentially offer light absorption enhancement exceeding the Lambertian limit.[8] The absorption enhancement by SPP waves in photovoltaic devices has been studied by several groups, but significant improvements in device performance has not been reported because the SPP waves can only enhance the absorption by a small amount—because (i) only incident p-polarized light (magnetic field perpendicular to the incidence plane) can excite SPP waves, and (ii) only one SPP wave can be excited at a specific free-space wavelength when the dielectric material is homogeneous normal to the interface.[1] In addition, the field is confined in a sub-wavelength region near the interface, leading to high absorption in the metal and limited enhancement in the active device region that is usually several wavelengths away from the interface.[9]

When the dielectric material is of finite thickness, the air/dielectric/metal structure can act as an open-face slab waveguide.[10,11] The fields of the waveguide modes are largely confined to the dielectric

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slab and depend on the thickness of that slab. These waveguide modes too can enhance the electric field, and therefore the electron-hole pair generation rate, in photovoltaic solar cells.[12–14]

When the dielectric material is periodically nonhomogeneous in the thickness direction, multiple SPP waves can be guided by its interface with the metal.[15,16] The excitation of the multiple SPP waves has been confirmed experimentally[17–19] over a broad range of incidence angles and wavelengths. Furthermore, the surface-multiplasmonics theory[15] shows that light absorption in thin-film solar cells can be increased by introducing periodic nonhomogeneity in the semiconductor material.[20] Finally, because the propagation length of some of these SPP waves in this structure is predicted to be as long as millimeters, the interface can be potentially exploited as a planar solar concentrator.[21]

Most often, experimental excitation of SPP waves requires either the use of a coupling prism or the periodic corrugation of the metal/dielectric interface.[15,22] For solar applications, the use of a prism[23] is impractical but the periodic corrugation of the interface has been investigated for three decades.[24,25]

In order to experimentally verify the impact of surface multiplasmonics on light absorption, we first deposited a periodically multilayered material—i.e., a one-dimensional periodic crystal (1D PC)—on a one-dimensional metallic grating and then measured the specular reflectance of this structure over broad ranges of the angle of incidence θ and free-space wavelength λ_0.[19] Our experimental findings were consistent with theoretical predictions. Most importantly, the same SPP wave at a certain λ_0 could be excited as Floquet harmonics of different order by light incident at different angles θ. Waveguide modes too can be excited as proficiently.[12,14,21]

If the 1D grating is replaced by a 2D grating, many more Floquet harmonics exist,[26] leading to increased possibilities of exciting multiple SPP waves and waveguide modes. Accordingly, guided-wave propagation can ocuur in several directions that are not confined in the incidence plane. Although theoretical formulation for rigorous solutions exists,[15] it requires computational resources that are still not easily available. Fortunately, in order to analyze experiments, full-scale theory is inessential and just a simple consequence of Floquet theory suffices. This has been experimentally demonstrated for multiple SPP waves when 1D[19] and 2D[27] gratings are used. But the demonstration did not address the waveguide modes that must have been simultaneously excited, and these waveguide modes would also play significant roles in enhancing the absorption of light by solar cells containing periodically nonhomogeneous semiconductors.

For this paper, we conducted an optical experiment on exactly the same structure (i.e., a 1D PC deposited on a 2D metallic grating) as in the predecessor paper[27] and analyzed the excitation of both multiple SPP waves and multiple waveguide modes. Complete details on the fabrication of that structure and the characterization of its constituent materials are already available.[19,27] Therefore, our focus here is only on the analysis of experimental data.

2. MATERIALS AND METHODS

A 2D grating of gold was fabricated with period $L = 350$ nm in both the x and y directions. Two periods of a 1D PC were deposited thereon. Each period of the PC comprises $N = 9$ layers of silicon oxynitride of different compositions $(SiO_2)_{a_j}(Si_3N_4)_{(1-a_j)}$, $a_j \in [0,1]$, $j \in [1,N]$. All materials are assumed to be isotropic. The measured refractive indexes of gold and all silicon-oxynitride layers are available elsewhere.[19] The thicknesses of the silicon-oxynitride layers are as follows: $d_j \in \{58, 60, 46, 59, 60, 60, 67, 71, 78\}$ nm, $j \in [1,N]$. A transmission-electron micrograph of the structure has been provided elsewhere.[27]

For all results presented here, the wave vector of the incident plane wave was taken to lie wholly in the xz plane, with $\theta \in [0°, 90°)$ measured with respect to the z axis, which is aligned in the thickness direction. The specular reflectance R_{s0} for incident s-polarized light and the specular reflectance R_{p0} for incident p-polarized light were measured on a custom-built apparatus[27] as functions of $\theta \in [8°, 45°]$ and $\lambda_0 \in [600, 1000]$ nm. The free-space wavenumber is denoted by $k_0 = 2\pi/\lambda_0$.

3. THEORY IN BRIEF

3.1 SPP wavenumbers

The SPP wavenumber q is obtained by solving a canonical boundary-value problem.[15, 16] Suppose the half space $z > 0$ is occupied by a metal and the half space $z > 0$ by the 1D PC. The interface can guide the propagation of an SPP wave parallel to the unit vector $\hat{\underline{u}}_{prop}$ such that $\hat{\underline{u}}_{prop} \bullet \hat{\underline{u}}_z = 0$, the associated fields varying in the xy plane as $\exp[iq\hat{\underline{u}}_{prop} \bullet (x\hat{\underline{u}}_x + y\hat{\underline{u}}_y)]$. The dependences of the fields on z are in consonance with the Floquet theory, but are too complicated to be discussed here.

Let us note that (i) q does not depend on the direction of $\hat{u}_{prop}$ since all materials are isotropic and the structure is transversely isotropic with respect to the z axis, and (ii) multiple values of q for any specific λ_0 are possible. As the procedure to determine q is described in detail elsewhere,[15, 16] we content ourselves here by providing the normalized wavenumbers q/k_0 as functions of λ_0 for the fabricated structure in Fig. 1. The obtained wavenumbers are organized in this figure into three branches for p-polarized SPP waves and two branches for s-polarized SPP waves.
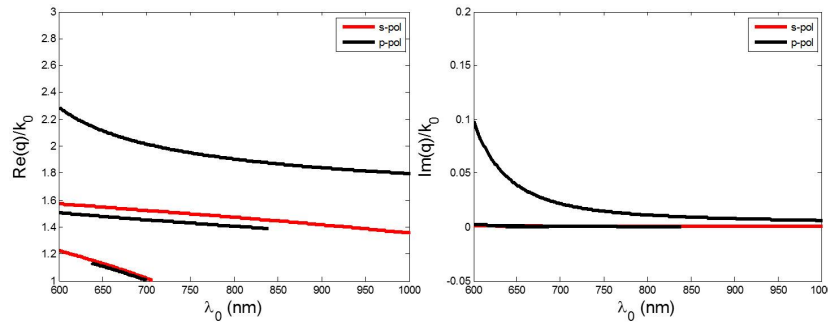

Figure 1. (a) Real and (b) imaginary parts of the normalized wavenumbers q/k_0 for SPP waves obtained after solving the canonical boundary-value problem. The waveumbers are organized into three branches for p-polarized SPP waves and two branches for s-polarized SPP waves.

3.2 Wavenumbers of waveguide modes

As the 1D PC is thicker than the free-space wavelength $\lambda_0 \in [600, 1000]$ nm, it supports multiple waveguide modes that trap light and play a very significant light-management role for solar-harvesting applications.[12–14, 21] The wavenumbers q of these modes can be obtained using a transfer-matrix formulation[28] that yields the following matrix equation for the structure under investigation:

$$
\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \left\{ \exp(i \left[\underline{\underline{P}}\right]_9 d_9) \bullet \exp(i \left[\underline{\underline{P}}\right]_8 d_8) \bullet \cdots \bullet \exp(i \left[\underline{\underline{P}}\right]_1 d_1) \right\}^2 \bullet \begin{bmatrix} \beta_p \, \alpha_0/k_0 \\ \beta_s \\ \beta_s \, \alpha_0/k_0\eta_0 \\ -\beta_p/\eta_0 \end{bmatrix}. \tag{1}
$$

Here, the 4×4 matrices

$$
\left[\underline{\underline{P}}\right]_j = \begin{bmatrix} 0 & 0 & 0 & \omega\mu_0 - q^2/\omega\epsilon_0 n_j^2 \\ 0 & 0 & -\omega\mu_0 & 0 \\ 0 & -\omega\epsilon_0 n_j^2 + q^2/\omega\mu_0 & 0 & 0 \\ \omega\epsilon_0 n_j^2 & 0 & 0 & 0 \end{bmatrix}, \quad j \in [1, 9]; \tag{2}
$$

$\omega = k_0 c_0$ is the angular frequency and $c_0 = 1/\sqrt{\epsilon_0\mu_0}$ is the speed of light in free space; ϵ_0 is the permittivity, μ_0 is the permeability, and $\eta_0 = +\sqrt{\mu_0/\epsilon_0}$ is the intrinsic impedance, of free space; $\alpha_0 = +\sqrt{k_0^2 - q^2}$ is
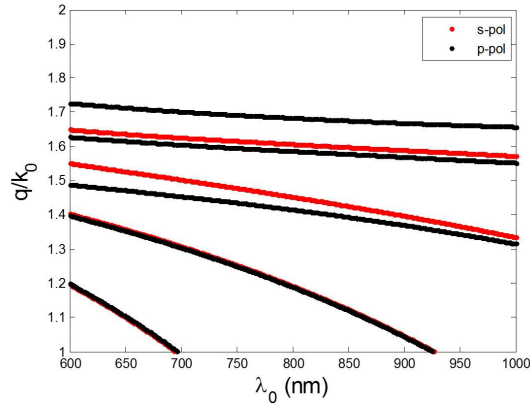
Figure 2. The normalized wavenumber q/k_0 for waveguide modes calculated using the transfer-matrix formulation for a 2-period PC bounded by free space and a perfect conductor. The wavenumbers are organized into five branches for p-polarized waveguide modes and four branches for s-polarized waveguide modes.

either positive real or positive imaginary; while β_p and β_s are finite constants. Furthermore, we have replaced gold by a perfect conductor, which is a reasonable simplification.

Equation 1 can be cast as an eigenvalue problem to yield the wavenumbers q of p- and s-polarized waveguide modes. For the fabricated structure, Fig. 2 provides the normalized wavenumbers q/k_0 of the waveguide modes. These wavenumbers are organized into five branches for p-polarized waveguide modes and four branches for s-polarized waveguide modes.

3.3 Predictions for a 1D PC atop a 2D metal grating

According to the Floquet theory, when a plane wave is incident on a finitely thick 1D PC atop a 2D metal grating, all fields must be decomposed everywhere in terms of Floquet harmonics. For the problem under consideration here, a Floquet harmonic of order (m, n) varies with respect to x and y and $\exp(ik_0 x \sin\theta) \cdot \exp[i2\pi(mx + ny)/L]$, where $m \in \{0, \pm1, \pm2, \ldots\}$ and $n \in \{0, \pm1, \pm2, \ldots\}$. An SPP wave or a waveguide mode with wavenumber q is excited as a Floquet harmonic of order (m, n), provided that

$$\mathrm{Re}(q) = \pm k_0 \sqrt{\left(\sin\theta + m\lambda_0/L\right)^2 + \left(n\lambda_0/L\right)^2}. \tag{3}$$

We inserted the values of q provided in Fig. 1 for SPP waves and in Fig. 2 for waveguide modes, and determined the triplets $\{\sin\theta, m, n\}$ that would satisfy Eq. 3 for each value of q. Of course, a triplet is considered as physically acceptable provided that $-1 < \sin\theta < 1$. The results are plotted in Fig. 3. It is possible for a specific SPP wave or waveguide mode to be excited as two different Floquet harmonics at the same value of λ_0. It is also possible for a SPP wave or waveguide mode of a specific polarization state to be excited by incident light of a different polarization state.

4. EXPERIMENTAL RESULTS

As stated in Sec. 2, we measured the specular reflectances R_{p0} and R_{s0} of the fabricated structure for $\theta \in [8°, 45°]$ and $\lambda_0 \in [600, 1000]$ nm. Given that the gold layer is sufficiently thick (150 nm), any transmittance is minuscule and nonspecular reflectances are null valued for the chosen ranges of θ and λ_0. Therefore, the corresponding absorptances of the structure can be calculated as $A_p = 1 - R_{p0}$ and $A_s = 1 - R_{s0}$. Experimental details can be found elsewhere.[27]

Figure 4 presents color maps of the absorptances A_p and A_s as functions of θ and λ_0. Higher absorptances are shown as reddish colors, lower absorptances as bluish colors. The color maps show broadband light absorption over the chosen range of θ, and we clearly observe reddish ridges of high absorptance
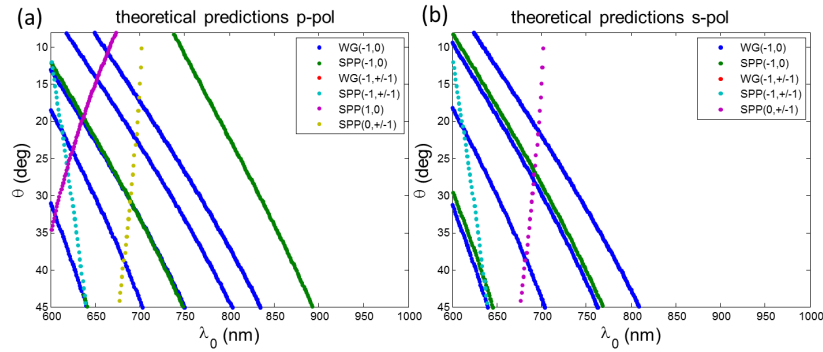
Figure 3. Predicted values of θ obtained from Eq. 3 and Figs. 1 and 2 for the experimental excitation of SPP waves and waveguide modes of (a) p polarization and (b) s polarization.
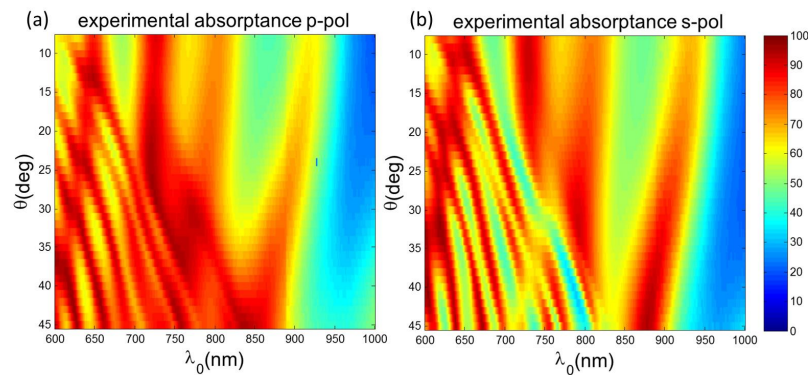

Figure 4. Color maps of the absorptances of the fabricated structure with the 1D PC of 2 periods for (a) p- and (b) s-polarized incident light.

corresponding to the excitation of SPP waves and waveguide modes. Overall, the high-absorptance ridges agree with the theoretically predicted curves in Fig. 3. However, one SPP wave is not observed in our experimental plots and the positions of some of the high-absorptance ridges are slightly shifted from our theoretical predicted curves. These discrepancies might be due to the distortion of the silicon-oxynitide layers in the fabricated structure. This is because these layers are coated conformally on the grating grating, and are not necessarily perfectly planar.

On comparing our theoretical and experimental results with those for the 1D grating,[19] we find that the 2D grating indeed leads to the excitation of more guided-wave modes. We observe broader-band light absorption (high absorptance for $\lambda_0 < 850$ nm) which is less sensitive to the incidence angle. The structure shows similar light coupling behavior for p- and s-polarized light indicating the absorption of the structure with a 2D grating is less dependent on the polarization state of the incident light. Also, a comparison with the predecessor paper[27] reveals that a rotation of the structure about the z axis will not greatly influence the absorption of light. Accordingly, the use of 2D gratings instead of 1D gratings should be more efficient for harvesting solar energy.

5. CONCLUDING REMARKS

The Floquet theory and the transfer-matrix approach were used to investigate the excitation of SPP waves and waveguide modes in a structure comprising a 1D PC of finite thickness on top of a planar thick metallic layer. The solutions of the relevant dispersion equations were used to predict the excitation of multiple SPP waves and waveguide modes when the metallic layer is patterned as a 2D surface-relief

grating. Dispersion equations of both the SPP waves and the waveguide modes were first calculated, and then the Floquet theory was applied to predict the experimental excitation of these guided-wave modes.

A two-period 1D PC was deposited on a gold grating and its absorptances for incident linearly polarized light were measured and mapped against the angle of incidence and the free-space wavelength, the wave vector of the incident light lying wholly in a plane defined by the thickness direction and one direction of periodicity of the metallic grating. Ridges of high absorptance in the maps agreed well with the theoretical predictions. Both the theoretical and experimental results demonstrated broadband coupling of light of any linear polarization state over a broad range of the angles of incidence. Thus, the fabricated structure can be potentially useful as a planar solar concentrator,[21] and our approach will be applicable also to thin-film solar cells.[20]

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