Modeling HIV Disease Progression with Partial Differential Equations

12/12/2023

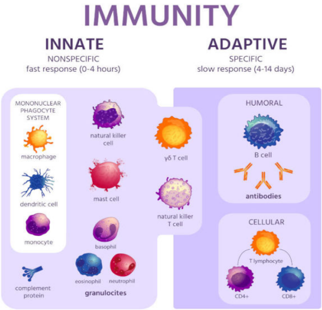
Sudhish Devadiga

Biological Mathematics Major

Mentored by: Xinxuan Wang

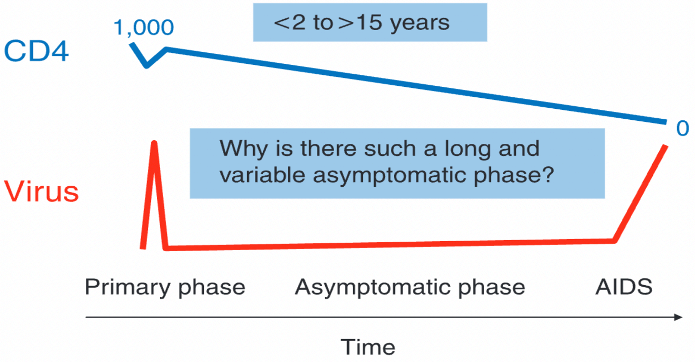
Penn
University of Pennsylvania

Introduction



- **HIV Targets CD4+ T-Cells!**

The clinical profile of HIV infection



Penn
University of Pennsylvania

Antigenic Variation Model

$$\begin{cases} \dot{v}_i = rv_i - px_iv_i \\ \dot{x}_i = cv_i - bx_i \end{cases} \tag{1}$$

$$i = 1, \ldots, n$$

v: amount of ith viral strain

x: amount of specific immunity against strain i

r: replication rate of virus

p: rate at which specific immune system kills virus

c: rate at which viral stimulation promotes specific immune response

b: rate at which immune cells decay

Penn
UNIVERSITY of PENNSYLVANIA

Strain-Specific and Cross-Reactive Immunity Model

- x: amount of specific immunity against strain i
- z: amount of innate immunity against virus
- q: rate at which innate immune system kills virus
- k: rate at which viral stimulation promotes innate immune response
- b: rate at which immune cells decay
- u: rate virus impairs immune system

$$\begin{cases} \dot{v}_i = rv_i - px_iv_i \\ \dot{x}_i = cv_i - bx_i \end{cases}$$

$$i = 1, \ldots, n$$

$$\begin{cases} \dot{v}_i = rv_i - px_iv_i - qzv_i \\ \dot{x}_i = cv_i - bx_i - uvx_i \\ \dot{z} = kv - bz - uvz \end{cases} \quad (2)$$

$$i = 1, \ldots, n$$

Penn
UNIVERSITY of PENNSYLVANIA

Change in Total Virus Population

$$x_i^* = \frac{cv_i}{b+uv}, i = 1, \ldots, n \quad (3)$$

$$z^* = \frac{kv}{b+uv} \quad (4)$$

$$\dot{v} = \frac{v}{b+uv}\left[rb - v(cpD + kq - ru)\right] \quad (5)$$

$$D = \sum_{i=1}^{n} \left(\frac{v_i}{v}\right)^2 \quad (6)$$

- b: rate at which immune cells decay
- r: replication rate of virus
- u: rate virus impairs immune system
- c: rate viral stimulation promotes specific immune response
- p: rate specific immune system kills virus
- k: rate viral stimulation promotes innate immune response
- q: rate innate immune system kills virus

Courses of Infection

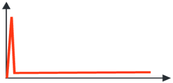
The evolutionary model
has 3 possible outcomes

1. Immediate disease

$kq + cp < ru$

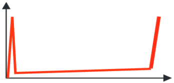
2. Indefinite virus control

$ru < kq$



3. Disease after long asymptomatic period

$kq < ru < kq + cp$



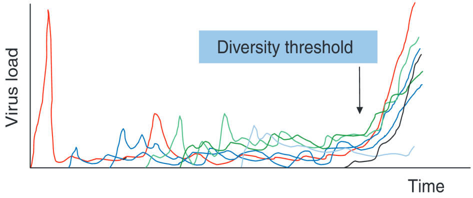
The Diversity Threshold

$$\dot{v} = \frac{v}{b + uv}\left[rb - v(cpD + kq - ru)\right]$$

$$D < \frac{ru - kq}{cp} \quad (7)$$

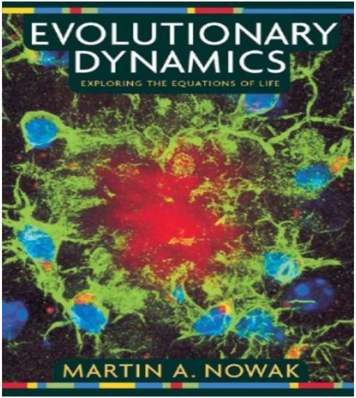
Evolution toward disease

- Escape from immune responses
- Faster replicating, more aggressive mutants
- Increased cell tropism



- ru: "power" of virus
- kq: "power" of innate immunity
- cp: "power" of strain-specific immunity

Reference


EVOLUTIONARY
DYNAMICS
EXPLORING THE EQUATIONS OF LIFE
MARTIN A. NOWAK

Penn
UNIVERSITY of PENNSYLVANIA