"Hard to Get, Easy to Lose" Endogenous Reputation and Underinvestment

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Abstract

Reputation concerns may help to achieve efficiency in cases where imperfect information impedes it. In this paper I highlight the endogenous property of reputation and analyze how it undermines the effectiveness of reputation to achieve efficiency. More reputation implies the possibility of undertaking more activity. At the same time, more activity generates more signals under which reputation can be constructed faster and more precisely.

Firms with low reputation have the incentives but not the demand to achieve efficiency. Firms with intermediate and high reputation may have the demand but not the incentives to achieve efficiency. High reputation firms have more room to lose reputation than to gain reputation. Covering the whole demand has short-term benefits from a large activity but has long-term costs from increasing the precision of signals and the magnitude of the potential loss of reputation, with the corresponding difficulty to build reputation back. The result is high reputation firms may decide to under-provide and under-invest.

1   Introduction

Reputation has been typically analyzed considering the problem of a single activity [1]. However reputation is inherently an endogenous problem. At the one hand, firms with reputation of selling high quality products, for example, have more demand and a wider consumer base than low reputation firms. In fact, there is a lot of empirical evidence that reputation increases the activity level [2].

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[1]Fudenberg and Levine (1992), Holmstrom (1999), Cripps, Mailath, and Samuelson (2004), Mailath and Samuelson (2001), Tadelis (2002), etc.

[2]Fishman and Rob (2005), Yoon, Guffey, and Kijewski (1993), Carmeli and Tishler (2005), and Antunovich, Laster, and Mitnick (2000)

At the other hand, the activity level also has an impact on the way reputation is constructed. A high activity level is characterized by many signals available to consumers in order to learn about the competence of the firm. In this sense, the higher the activity level, the more precise and faster the learning process about the firm's type.

Hence, not only reputation determines the activity level but also the activity level determines how reputation is formed. This paper combines these two effects in a single framework.

The introduction of endogenous reputation helps to understand decisions by a firm that seem strange at first sight such as, why firms underprovide services even when having the opportunity of expanding their consumer base? Why firms offer new products under cost or even for free? Why managers that are partially paid with firm stocks invest less than managers whose payments depend on sales or activity levels?

First, firms may decide to underprovide goods even if the demand for those goods exist and the firm is able to extract the whole consumer surplus. This will happen when undertaking an additional activity exerting efforts to maintain reputation is costly, but undertaking it without exerting efforts may decrease reputation more than raising benefits.

Second, when a competent firm has a low reputation (hence low activity level) may find beneficial to lose some money in the present in order to create more signals to gain reputation faster for the future.

Third, managers that are paid based on sales will try to undertake all possible activities, since they do not care about the firm's reputation. However, managers that are paid with firm's stocks will care about its reputation, underproviding when optimal, as explained above.

Additionally to answering these questions we will analyze the difficulties to achieve efficiency under endogenous information. When reputation is low, consumers will demand less from the firm than what would be efficient if knowing its type. The firm will undertake all activities and exert efforts in all of them. When reputation is intermediate, the firm will underprovide but exerting efforts in all of them. Finally, when reputation is high, the firm will not only underprovide but will also exert efforts only on some activities.

Hence, to achieve efficiency will be in general difficult when accepting the possibility that reputation is an endogenous process. A firm with reputation concerns may decide to optimally underprovide, which clearly contrasts with the efficiency results obtained by the typical models with perfect discrimination.

Section 2 presents the model. Section 3 describes endogeneity and efficiency in the model. Section 4 discusses incentives under reputation concerns and the possibility of achieving efficiency in an endogenous environment. Section 5 shows a numerical exercise to make the intuition more transparent. Finally, Section 6 concludes.

2 The Model

2.1 Description

Assume a monopolist whose activity is the provision of an homogeneous experience service or good to different groups of consumers, charging the expected utility to each group since it's able to perfectly discriminate among them.[3]

The firm can be one of two possible types, Competent (*C*) or Inept (*I*). Both competents and inepts decide the quantity of groups n to serve out of n_D clients who demand the service (defining its activity level) but only competents can decide whether or not to exert high efforts in serving a group i[4]. The total number of groups where competents decide to exert high efforts will be denoted by n_h. While serving a group i has a provision cost f_i, exerting high efforts additionally costs c_i.

After the decision is made, production happens and a non-deterministic output, which can be a success or a failure in each group, is obtained. When competents exert high efforts, the probability of a success is $\alpha > 1/2$. When competents exert low efforts, the probability of a success is $\beta < \alpha$.

Finally, at the end of the period, the firm's owner may be replaced by another one with a fixed probability λ. The substitute will be competent with a probability $\theta \in (0, 1)$ [5]

Consumers are divided into *N* groups (such that in each group there is a continuum of identical persons of unit mass where no single individual can affect the future play of the game). Groups differ in the value assigned to the good or service. Group i values a successful result by $u(g) = z_i$, where $z_i \in [0, 1]$, and a failure by $u(b) = 0$.

Consumers cannot see the effort choice of the monopolist (moral hazard), nor its type (adverse selection). They can only see the results from production activities in all the groups the monopolist provided the good or service. From this information, they update the probability that the monopolist is competent, $Pr(C) = \phi$, (i.e. his or her reputation). This is of the utmost importance to the firm since we assume each group has to buy the good or service before production takes place, hence paying the expected utility and not the real utility it delivers.

The greater the reputation (probability of the firm being competent), the greater the probability assigned by consumers to obtain good outcomes. If this happens, all groups would be

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[3]It may be useful to think as an example on water or electricity companies that sell the same service to heterogenous towns or cities.

[4]Another example is competents who can make unobservable investments to produce higher quality while inepts cannot do that.

[5]This assumption is needed to sustain an efficient equilibrium in the long run when serving just one group, as discussed in Mailath and Samuelson (2001) and Cripps, Mailath, and Samuelson (2004).

willing to pay more for the good or service and more groups who would be willing to demand the good. This is the reason monopolists are so concerned about reputation while consumers are only concerned about the utility derived from buying the good.

2.2 Timing

The timing of the model is:

1) Based on its reputation, the firm decides how many groups (n^*) to serve by selling the good or providing the service.

2) At the beginning of the period, before the production takes place, the firm receives the payment from each served group, which only depends on its reputation and not on the period's true type, efforts exerted or real production result.

3) Competents decide between exerting high or low efforts in each group. Inepts can only exert low efforts in selling to all groups . Consumers do not observe these decisions.

4) Output is produced and both consumers and the firm observe the true utility given by a success or a failure to all groups served. All consumers in a given group receive the same realization of utility outcome, which is public to all groups.

4) With probability λ the firm's owner is replaced by another one, who is competent with a probability θ.

2.3 Equilibrium Definition

In the presence of uncertainty about the firm's type, the state variable is just the probability assigned by all consumers' groups to the firm being competent (i.e. the reputation ϕ).

A Markov strategy for competents is a mapping $\tau_i(\phi) : [0,1] \rightarrow [0,1]$, where $\tau_i(\phi)$ is the probability of exerting high efforts in group i when reputation is $\phi \in [0,1]$. Inepts make no choice, having then a trivial strategy of exerting low effort. [6]

Both competents and inepts have as a Markov strategy n (the number of groups to serve) which is a mapping $n(\phi) : [0,1] \rightarrow [0, n_D(\phi)]$ where n_D is the existent demand at each level of reputation $\phi \in [0,1]$.

The behavior of each group i of consumers is described by the Markov belief function p : $[0,1] \rightarrow [0,1]$ where $p_i(\phi) = Pr(g|\phi)z_i$ is the expected utility from buying the good. Naturally, $Pr(g|\phi)$ depends positively on the reputation of the firm $\phi \in [0,1]$.

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[6]As noted in Mailath and Samuelson (2001), by restricting attention to strategies that only depend on consumers' posteriors, in equilibrium different firms will behave identically in identical situations.

In a Markov perfect equilibrium firm maximizes profits, consumers' expectations are correct and consumers use a Bayes' rule to update their posterior probabilities.

Since the state variable is the reputation ϕ, the model relies importantly on the updating of beliefs about the competence of the monopolist. Exerting low efforts increase profits today by reducing costs but it is harmful for reputation. Serving many groups increase profits today but makes the updating of beliefs more precise.

To show this explicitly in terms of parameters and decision rules, the updating after each possible combination of groups served (n) and number of successes (s) is,

$$Pr(C|s,n) = \frac{Pr(s|C,n)\phi}{Pr(s|C,n)\phi + Pr(s|I,n)(1-\phi)} \tag{1}$$

where

$$Pr(s|C,n) = \alpha^s(1-\alpha)^{n-s}\frac{\sum_{i=1}^n \tau_i}{n} + \beta^s(1-\beta)^{n-s}\frac{\sum_{i=1}^n(1-\tau_i)}{n}$$

$$Pr(s|I,n) = \beta^s(1-\beta)^{n-s}$$

Before defining the equilibrium it is necessary to define the value function for the firm. Considering it decides to provide services to n groups and to exert high efforts in n_h of them [7], for a given value of ϕ, the value function will be given by,

$$V(\phi; n, n_h) = p(\phi, n) - c(n_h) + \delta(1-\lambda)[\sum_{s=0}^n Pr(s|n, n_h)V(\phi_{s,n})] \tag{2}$$

where $p(\phi, n) = \sum_{i=1}^n [p_i(\phi) - f_i]$ and $c(n_h) = \sum_{i=1}^{n_h} c_i$

Definition 1 *A Markov perfect equilibrium* [8] *is, for a reputation prior $\phi = Pr(C)$: Number of groups served $n^*(\phi)$, probabilities of exerting high effort $\tau_i(\phi)$ in each served group, payments consumers are willing to make for receiving a good outcome $p_i(\phi)$, and posterior beliefs $\varphi = Pr(C|s,n,\phi)$ (where s is the number of successes), such that:*

1) Activity level (n) and efforts exerted by competent firms:

$n(\phi)$ and $\tau_i(\phi)$ maximize the value function $V(\phi)$ for all possible reputation values ϕ

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[7]This is the same than assuming the probability of exerting high efforts in each group is $\tau_i(\phi) = \frac{n_h}{n}$

[8]We require behavior to be Markov in order to eliminate equilibriums that depend on implausible degrees of coordination between the firm behavior and consumers belief's about the firm behavior. (See discussion in Mailath and Samuelson (1998))

2) Expected utility (and payments) for each served group i:

$$p_i(\phi) = Pr(g|\phi)z_i = [(\alpha\tau_i(\phi) + \beta(1 - \tau_i(\phi)))\phi + \beta(1 - \phi)]z_i \tag{3}$$

3) Beliefs about competence (updated using Bayes rules):

$$\varphi(\phi|s, n) = \phi_s^n = (1 - \lambda)Pr(C|s, n) + \lambda\theta \tag{4}$$

A strategy for the monopolist uniquely determines the equilibrium updating rule that "consumers" must use if their beliefs are to be correct.

3 Endogenous information

For each value ϕ we can obtain the optimal n and n_h that maximizes $V(\phi)$. However, in this work we are interested in characterizing the sufficient conditions for an efficient equilibrium.

In order to do this, we will define efficiency formally and then we will analyze the sufficient conditions to achieve efficient results in equilibrium.

3.1 Efficiency

It's important to obtain the efficient level of activity and effort in order to analyze incentives in an environment of endogenous formation of reputation.

Assume there are N potential groups interested in the good. Each group values the good by z_i. Assume $z_1 = 1$, $z_N = 0$ and $z_i' < 0$. Since by exerting high efforts the probability of a success is α, the demand for a "high effort good" will be given by αz_i. Similarly, the demand for a "low effort good" will be given by βz_i. We will not make at this point any assumption about how provision costs (f_i) and effort costs (c_i) vary across groups i.

For simplicity in the next Propositions we will assume N is big enough so we can approximate a continuum result. Otherwise we should need to deal with having integers in the solution.

Proposition 1 *Efficiency*

Denoting n^E the efficient level of total groups served and n_h^E the efficient number of those groups where it's optimal to exert high effort,

1) If $c(n^E) \geq \frac{\alpha - \beta}{\beta} f(n^E)$, then

$$n^E = z^{-1}\left[\frac{f(n^E)}{\beta}\right]$$ (5)

$$n_h^E = z^{-1}\left[\frac{c(n_h^E)}{\alpha - \beta}\right]$$ (6)

2) If $c(n^E) < \frac{\alpha - \beta}{\beta} f(n^E)$, then

$$n^E = n_h^E = z^{-1}\left[\frac{f(n^E) + c(n^E)}{\alpha}\right]$$ (7)

This means that for effort costs (c) low enough compared with provision costs (f) at the optimum, it will be efficient to exert high efforts in all the served groups. Contrarily, when c is high enough compared with f it will be optimal to exert low effort on some of the served groups. The proof in the Appendix.

The next corollary shows the same result for the case in which the demand is linear and provision and effort costs are fixed. This will be important as a benchmark for the numerical exercise in Section 5.

Corollary 1 *Efficiency under linear demand and constant costs*

Assuming $\alpha z_i = \alpha(1 - \frac{i}{N})$, $\beta z_i = \beta(1 - \frac{i}{N})$, $f_i = \overline{f}$ and $c_i = \overline{c}$, efficient results n^E and n_h^E are,

1) If $\overline{c} \geq \frac{\alpha - \beta}{\beta}\overline{f}$, then

$$n^E = \frac{N}{\beta}[\beta - \overline{f}]$$

$$n_h^E = \frac{N}{\alpha - \beta}[\alpha - \beta - \overline{c}]$$

2) If $\overline{c} < \frac{\alpha - \beta}{\beta}\overline{f}$, then

$$n^E = n_h^E = \frac{N}{\alpha}[\alpha - \overline{f} - \overline{c}]$$

The results in this corollary can be obtained easily from the proof of Proposition 1 just considering $z_i = 1 - \frac{i}{N}$

3.2 Endogeneity

This setup allows to rationalize the endogeneity of firm's activity. Assume the firm does not have access to external finance or cross subsidies among activities. We will relax these
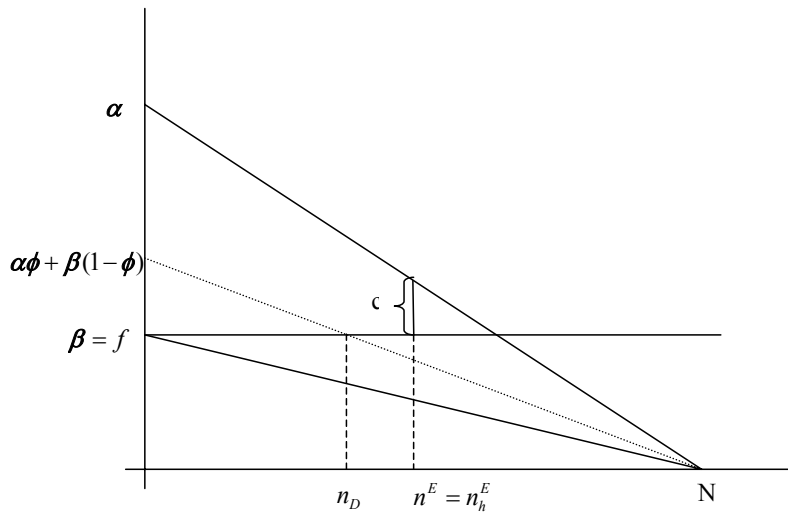
assumptions later in order to show the impact on reputation formation. Assume also both provision costs and the outside option for each group are constant and equal to β. This is the minimum the firm can charge for the good or service. This assumption guarantees that, when groups are completely sure the monopolist is inept, only the first one (that values the service a lot $z_1 = 1$) will be indifferent between buying the good or not.[9]

As the reputation increases, groups that value the service more start to demand it since the expected value (given by $(\alpha\phi + \beta(1 - \phi))z_i$) will be greater than $f = \beta$ for some of the groups.

Naturally the same analysis holds for provision costs $f < \beta$ in which case some groups would be willing to buy, even being sure the firm is inept. Contrarily, when $f > \beta$ it will be necessary some minimum amount of reputation before the monopolist have some demand at all.

Analyzing Figure 1 may be useful to understand the structure of the demand and the importance of reputation for the firm, both to obtain more profits from each group and more groups to work with.

Figure 1: Reputation varying demand

The graph represents a typical demand function. The expected value of a service provided by a competent who exerts high efforts is greater than the expected value of a service provided by an inept (or a competent who exerts low efforts). For example, in the first case, the service delivers an expected value of α to the first group ($z_1 = 1$) since α is the probability of getting a success (and a utility equal to 1). Contrarily, in the second case the service delivers an

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[9]Another interpretation is that each group can always get a value of β buying outside or self generating the product. If the group decides to buy to the monopolist he must incur in transportation or transaction cost that differ across groups and generates the function z_i described above. Under this interpretation, groups differ on the net value assigned to the good or service from the monopolist, a kind of "home bias" effect.

expected value equal to β.

At the other side of the picture, the group N (who assign a value $z_N = 0$ to the service) does not care who produces it. However, when the group does not know the monopolist's type and just believe there is a probability ϕ the firm is competent, the first group will value the good in expected terms by $\alpha\phi + \beta(1 - \phi)$ represented by the dotted line.

As reputation increases, not only each group is willing to pay more for the good but new groups start to demand as well.

4   Incentives under endogenous information

In this section we will analyze how incentives behave under endogenous information. As explained above, low reputation not only means it's difficult to extract surplus from clients but also there are no many clients from which surplus can be extracted.

The activity level does not only have an effect on the revenues but also on the speed at which reputation can be constructed. In this section we will focus on the existence of the efficient equilibrium in which the monopolist exerts high efforts in the n^E groups obtained in Proposition 1 (i.e., we will assume provision costs f are high enough such that in the efficient situation $n_h^E = n^E$).

Definition 2 *Classification of reputation*

Low reputation: $\phi_{LR} \in [0, \phi_1]$ where ϕ_1 fulfills the following condition

$$[E_\phi^{n_D, n_D} - E_\phi^{n_D-1, n_D-1}] = \frac{[c_{n_D} - c_{n_D-1}] - [p_{n_D} - p_{n_D-1}]}{\delta(1-\lambda)(\alpha - \beta)} \tag{8}$$

where $E_\phi^{a,b}$ is the expected reputation when serving a groups and exerting high efforts in b of them. n_D is the number of groups that demand the service at a given reputation level ϕ.

Intermediate reputation: $\phi_{IR} \in (\phi_1, \phi_2]$ where ϕ_2 fulfills the following condition

$$[E_\phi^{n+1, n} - E_\phi^{n,n}] = \frac{[p_{n+1} - p_n]}{\delta(1-\lambda)(\alpha - \beta)} \tag{9}$$

where n is the number of groups served at a given reputation level ϕ

High reputation: $\phi_{HR} \in (\phi_2, 1]$

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The conditions that determine parameters ϕ_1 and ϕ_2 in Definition 2 will arise endogenously later from solving the model. Expectations above are calculated considering consumers believe that competents always exert high efforts in all served groups (i.e., $n_h = n$ or $t_i = 1$ for all $i \in 1, .., n$) This is based on the objective of checking the conditions for the efficiency to arise as an equilibrium.

There are two sources of inefficiencies in this equilibrium. One arises when competents decide not to exert high effort in some of the groups served. The other arises when the firm decides to underprovide the service (i.e. to serve less groups than possible and efficient) even when making high efforts on the served ones.

Next propositions depict the incentives in each of the defined intervals. Proofs can be found in the Appendix.

Proposition 2 *Incentives under low reputation*

If the firm has a low reputation (in the interval ϕ_{LR}), it will provide the service to all groups who demand it ($n = n_D$) and will exert high efforts on all of them ($n_h = n$).

The intuition of this proposition is simple. When reputation is low, the expected gains in reputation from exerting high efforts are big. Furthermore, the greater the activity level, the faster the increase in reputation. Hence, the firm would like to exert high efforts in many groups since this guarantees a fast increase in reputation. The problem is that when reputation is low the demand restricts the possible activity level.

Considering this reasoning, the optimal decision for the firm is to serve all possible groups and exert high effort in all of them. Naturally, if the expected gains in reputation are smaller than effort costs, then $\phi 1 = 0$, and the low reputation interval defined as above just does not exist.

Proposition 3 *Incentives under intermediate reputation*

If the firm has an intermediate reputation (in the interval ϕ_{IR}), it will underprovide the service ($n < n_D$) and will exert high efforts on all of the served groups ($n_h = n$).

For intermediate reputation levels, the expected reputation gains are not so important to justify the exertion of high efforts. However, the expected reputation loses from serving an additional group without exerting efforts are greater than the surplus it's possible to extract from that additional group. Hence, it's optimal for the firm to give up some activity to maintain the reputation.

In this interval consumer's beliefs are correct but the inefficiency arises because of the optimal underprovision.

Proposition 4 *Incentives under high reputation*

If the firm has a high reputation (in the interval ϕ_{HR}), it will underprovide the service ($n < n_D$) and will exert high efforts only on some of the served groups ($n_h < n$).

When the firm has a high reputation, the expected reputation gains from exerting high efforts are very low and the expected reputation loses from providing an additional group without exerting efforts are also low

In this interval it's better for the firm to cheat on the consumers because there is no incentive to exert high efforts in all served groups. Hence consumers' beliefs that firm will always exert high efforts is not an equilibrium.

In this interval coexist both types of inefficiencies. The firm prefers to serve more groups (since from the last interval the demand is greater than the groups effectively served) and exert less effort among the served groups.
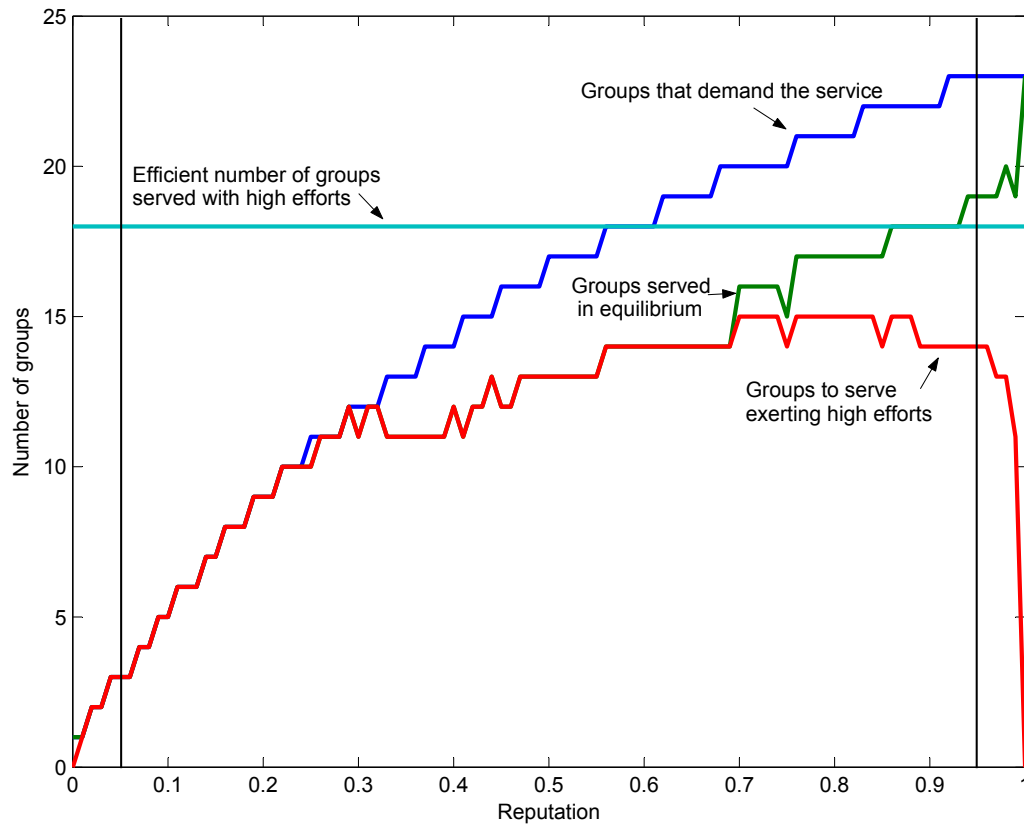
5   Simulations

To fully solve this model analytically, one would have to write out a function that took in each possible reputation ϕ and produced the expected reputation in the following period. However, the changing number of signals complicates the writing of a general function for every possible ϕ (i.e. there would have to be a separate function for each one). Since most markets involve a large number of potential groups, writing out an explicit solution is intractable.

In this section we make an exercise that allows to observe the basic forces behind the idea of underprovision due to reputation concerns and endogenous formation of beliefs. We use the following parameters. $N = 40$, $z_i = (1 - \frac{i}{N})$, $\alpha = 0.7$, $\beta = 0.3$, $\overline{f} = 0.3$, $\overline{c} = 0.1$, $\delta = 0.9$, $\lambda = 0.1$ and $\theta = 0.5$. This case corresponds to the special case described in Corollary 1. In particular $\overline{c} < \frac{\alpha - \beta}{\beta}\overline{f}$, which means it is efficient that $n^* = n_h^* = \frac{N}{\alpha}[\alpha - \overline{f} - \overline{c}] \simeq 18$ [10]

Figure 2 shows how groups that demand the service positively depend on the reputation level. When reputation is low the firm undertakes the whole possible activity, exerting high efforts in all of the groups where the service is provided.

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[10]Recall we're using discrete and indivisible groups so the condition is not strictly an equality. Efficiency is represented by the "Efficient number of groups served with high efforts" in Figure 2

Figure 2: Underprovision under endogenous reputation. An exercise

When reputation is intermediate, the firm optimally decides to underprovide, not taking all the possible activities but exerting high efforts in all of them. In Figure 2, the curve representing the number of groups served in equilibrium start to differ from the demand but not from the number of groups where high efforts are exerted.

Finally, when reputation is high we have the typical effect of reputation in reducing the incentives to exert efforts, but magnified by the number of signals. In Figure 2, the curves representing the number of groups served and the number of groups with high efforts start to differ in equilibrium.

As can be seen, the lack of information about the type of the firm impedes efficiency from a demand point of view since consumers demand less activity than socially efficient given the lack of knowledge about the firm's type. This problem is more important the less the reputation of the firm.

When the firm has an enough level of reputation for consumers to demand the efficient activity level, the firm may optimally decide to underprovide in order to maintain the reputation.

Finally, when reputation is high enough for the society to demand the efficient level and for firms to provide the service, it will not be the best for the monopolist to exert high efforts in an efficient way.

As can be seen, under endogenous reputation it may be very difficult to achieve efficiency. Naturally this conclusion depends on the parameters. For example, if $\bar{c} = 0$, efficiency would arise in equilibrium. Hence, it exists a $\bar{c}$ low enough that leads to efficiency. However the $\bar{c}$ that leads to equilibrium is very low when compared with a constant information case.

6   Conclusions

This paper highlights the endogenous property of reputation. We develop a model where both the reputation increases the level of activity (which is a characteristic of reputation widely cited in the empirical business literature) and the level of activity has an effect on the formation of reputation. The more the activity level, the more the number of signals that determine reputation, allowing for a faster and easier learning process.

Considering this double way of reactions, we show the difficulties to achieve efficiency in equilibrium because of the underprovision of activity as an optimal policy of the firm.

Endogenous reputation also helps to understand different firm's decisions, such as the provision of new products below costs in order to spread signals to build reputation faster, or the payment to managers using stocks rather than bonuses based on sales, in order to internalize reputation effects.

This model can eventually help us to understand the formation of teams or the separation of activities under intermediate reputation. The intuition is that firms may want to reduce the precision of learning when reputation is high.

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A Appendix

Proof of Proposition 1

The social planner would maximize the consumer surplus such that

$$max_{n,n_h} \int_0^{n_h} [\alpha z_i - f_i - c_i]di + \int_{n_h}^{n} [\beta z_i - f_i]di \tag{10}$$

subject to $n \geq n_h$. Since $f_i + c_i$ is assumed to be positive, it's a trivial result that $n < N$.

From the lagrangian and assuming interior solutions, the first order conditions are

$$n: \quad (\beta z_{n_h} - f_n) + \lambda = 0$$
$$n_h: \quad (\alpha - \beta)z_{n_h} - c_{n_h} - \lambda = 0$$
$$\lambda: \quad n \geq n_h \quad [= 0 \text{ if } \lambda > 0]$$

and from these equations the results shown in the proposition follow easily   Q.E.D.

Proof of Proposition 2

In order to prove this proposition we will start analyzing what is optimal to do for a firm that is serving n firms and have an increase in demand of one additional group. This will happen only after an increase in reputation which immediately implies it's not optimal to reduce the activity (since $p(\phi, n)$ increases, raising $V(\phi)$).

Hence we can make the analysis from the lowest possible reputation level $\phi = \lambda\theta$. Assume the following condition fulfills $V(1,1) \geq V(1,0)$. This means the value of exerting high efforts when the firm serves one firm is greater or equal than the value of not exerting any effort.

If the sufficient condition

$$[E_\phi^{1,1} - E_\phi^{1,0}] \geq \frac{c_1}{\delta(1 - \lambda)(\alpha - \beta)} \tag{11}$$

holds for all feasible $\phi \in [\lambda\theta, 1 - \lambda(1 - \theta)]$, then $V(1,1) \geq V(1,0)$ [11]

This assumption guarantees the firm will efficiently exert high efforts when serving just one firm. Assume the firm has so low reputation that only one group demands the service and, given the previous assumption, the firm exerts high efforts in that group.

Now consider reputation raises such that two groups demand the service. The firm needs to decide among three choices. Serve two groups and exert high effort in both, serve two groups but just exert high efforts in one or maintaining the current activity in one group.

The firm will choose the first option if two conditions hold,

$$[E_\phi^{2,2} - E_\phi^{1,1}] \geq \frac{[c_2 - c_1] - [p_2 - p_1]}{\delta(1 - \lambda)(\alpha - \beta)}$$

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[11] This is the same condition exposed in Mailath and Samuelson (2001), who consider the firm can just serve one group

$$[E_\phi^{2,2} - E_\phi^{2,1}] \geq \frac{[c_2 - c_1]}{\delta(1-\lambda)(\alpha-\beta)}$$

The first condition means it should be better to serve the second group and exert high effort in it rather the maintaining the current activity. The second condition means it's better to exert high efforts in the second group rather than low efforts in case of provide the service to that group.

In general, if the firm was serving n groups the conditions are

$$[E_\phi^{n+1,n+1} - E_\phi^{n,n}] \geq \frac{[c_{n+1} - c_n] - [p_{n+1} - p_n]}{\delta(1-\lambda)(\alpha-\beta)} \tag{12}$$

$$[E_\phi^{n+1,n+1} - E_\phi^{n+1,n}] \geq \frac{[c_{n+1} - c_n]}{\delta(1-\lambda)(\alpha-\beta)} \tag{13}$$

These conditions will hold in the low reputation interval because the increase in expected reputation is big. In words, the discounted expected reputation is greater or equal than the net cost of exerting high efforts.

Some characteristics of the conditions are worth noting. First, the left hand side $[E_\phi^{n+1,n+1} - E_\phi^{n,n}]$ is big for low n and is decreasing in n. Second, the difference $[p_{n+1} - p_n]$ is close to zero when the group $n+1$ just starts to demand since the surplus is just becoming positive. Finally, $[E_\phi^{n+1,n+1} - E_\phi^{n,n}] \leq [E_\phi^{n+1,n+1} - E_\phi^{n+1,n}]$ for all n.

Under these considerations, as the left hand side decreases with n and the right hand side stays roughly constant [12], the condition under which ϕ_1 is obtained in Definition 2 arises.

In fact, ϕ_1 is obtained only when equation 12 holds with equality because the left hand side of equation 12 is always greater than the the left hand side of equation 13 while the right side of both equations are basically the same.

Hence, whenever $\phi_{LR} \in [0, \phi_1]$, to serve the whole demand $n = n_D$ and to exert high efforts in all those groups $n_h = n$ is an equilibrium.

Q.E.D.

Proof of Proposition 3

In order to prove this proposition we will start analyzing what is optimal to do for a firm that is serving n firms and have an increase in demand of one additional group just at the beginning of the second interval as defined in Definition 2.

Since the interval for intermediate reputation has been defined such that $V(n+1, n+1) \leq V(n,n)$ (opposite sign than condition 12), it's better to continue serving the same quantity of groups rather than increasing the activity.

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[12]This is also true as effort costs increase or even when they decrease at a rate smaller than the decrease in expected reputation.

The question is whether it's better to just maintain the activity level or to serve the additional group without exerting efforts on it. The answer will depend on the sign of $V(n, n) - V(n + 1, n)$. If it's better to maintain the activity level, the sufficient condition

$$[E_\phi^{n,n} - E_\phi^{n+1,n}] \geq \frac{[p_{n+1} - p_n]}{\delta(1 - \lambda)(\alpha - \beta)} \tag{14}$$

should hold. In fact, this condition is very likely to hold at the beginning of the interval since $[p_{n+1} - p_n]$ is close to zero. As $[p_{n+1} - p_n]$ increases, the firm starts to serve more groups, but the same logic applies.

This interval ends (condition 14 does not hold anymore) whenever $\phi > \phi_2$ as defined in Definition 2.

<div align="right">Q.E.D.</div>

Proof of Proposition 4

The condition that will be fulfilled in this interval is $V(n + 1, n) \geq V(n, n)$ (by definition 2 and condition for ϕ_2).

Now, the question is whether it's optimal to exert high efforts in the new served group or not. The answer will depend on the sign of $V(n + 1, n) - V(n + 1, n + 1)$. If it's better to exert high efforts in the new served group, the sufficient condition

$$[E_\phi^{n+1,n+1} - E_\phi^{n+1,n}] \geq \frac{[p_{n+1} - p_n]}{\delta(1 - \lambda)(\alpha - \beta)} \tag{15}$$

should hold. However, this condition is hard to hold since expected reputation gains are low and at this point the difference between the number of group served and the demand is high enough such that the marginal group has a lot of surplus to extract.

<div align="right">Q.E.D.</div>