

# A Prototypical DSGE Model

**Frank Schorfheide**

Department of Economics, University of Pennsylvania

## A Prototypical DSGE Model

- Detailed discussion provided in Woodford (2003).
- Households: consume, competitively supply labor services, value transaction services from real money balances,
- Final Good Producers: aggregate a continuum of intermediate goods.
- Intermediate Goods Producers: hire labor services, AR(1) technology. Monopolistic competition and adjustment costs for nominal price changes.
- Monetary Policy: represented through interest rate feedback rule.

## A Prototypical DSGE Model

- Final good production:

$$Y_t = \left( \int_0^1 Y_t(j)^{1-\nu} dj \right)^{\frac{1}{1-\nu}}. \quad (1)$$

- Demand for intermediate inputs:

$$Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-1/\nu} Y_t. \quad (2)$$

- Price of final good:

$$P_t = \left( \int_0^1 P_t(j)^{\frac{\nu-1}{\nu}} dj \right)^{\frac{\nu}{\nu-1}}. \quad (3)$$

## A Prototypical DSGE Model

- Intermediate goods are produced by monopolistically competitive firms with the following technology:

$$Y_t(j) = A_t N_t(j), \quad (4)$$

- $A_t$  is an exogenous productivity process that is common to all firms and  $N_t(j)$  is the labor input of firm  $j$ .
- Labor is hired in a perfectly competitive factor market at the real wage  $W_t$ .
- Firms face nominal rigidities in terms of quadratic price adjustment costs

$$AC_t(j) = \frac{\varphi}{2} \left( \frac{P_t(j)}{P_{t-1}(j)} - \pi \right)^2 Y_t(j), \quad (5)$$

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- Firm  $j$  chooses its labor input  $N_t(j)$  and the price  $P_t(j)$  to maximize the present value of future profits

$$\mathbb{E}_t \left[ \sum_{s=0}^{\infty} \beta^s Q_{t+s} \left( \frac{P_{t+s}(j)}{P_{t+s}} Y_{t+s}(j) - W_{t+s} N_{t+s}(j) - AC_{t+s}(j) \right) \right]. \quad (6)$$

- The household maximizes

$$\mathbb{E}_t \left[ \sum_{s=0}^{\infty} \beta^s \left( \frac{(C_{t+s}/A_{t+s})^{1-\tau} - 1}{1-\tau} + \chi_M \ln \left( \frac{M_{t+s}}{P_{t+s}} \right) - \chi_H H_{t+s} \right) \right], \quad (7)$$

- subject to the budget constraint

$$P_t C_t + B_t + M_t - M_{t-1} + T_t = P_t W_t H_t + R_{t-1} B_{t-1} + P_t D_t \quad (8)$$

## A Prototypical DSGE Model

- Monetary policy is described by an interest rate feedback rule of the form

$$R_t = R_t^*{}^{1-\rho_R} R_{t-1}^{\rho_R} e^{\epsilon_{R,t}}, \quad (9)$$

- Output gap rule:

$$R_t^* = r\pi^* \left( \frac{\pi_t}{\pi^*} \right)^{\psi_1} \left( \frac{Y_t}{Y_t^*} \right)^{\psi_2} \quad (\text{output gap rule specification } \mathcal{M}_1) \quad (10)$$

- Output growth rule:

$$R_t^* = r\pi^* \left( \frac{\pi_t}{\pi^*} \right)^{\psi_1} \left( \frac{Y_t}{\gamma Y_{t-1}} \right)^{\psi_2} \quad (\text{output growth rule specification } \mathcal{M}_2). \quad (11)$$

- Government budget constraint

$$P_t G_t + R_{t-1} B_{t-1} = T_t + B_t + M_t - M_{t-1}. \quad (12)$$

## A Prototypical DSGE Model

- Exogenous government spending:

$$G_t = \zeta_t Y_t \quad (13)$$

- Resource constraint

$$Y_t = C_t + G_t = C_t + \zeta_t Y_t \implies Y_t = \frac{1}{1 - \zeta_t} C_t \quad (14)$$

- Take logs and define  $g_t = 1/(1 - \zeta_t)$ :

$$\ln Y_t = \ln g_t + \ln C_t \quad (15)$$

## A Prototypical DSGE Model

- Exogenous processes...
- Technology evolves according to

$$\ln A_t = \ln \gamma + \ln A_{t-1} + \ln z_t, \quad \text{where} \quad \ln z_t = \rho_z \ln z_{t-1} + \epsilon_{z,t}. \quad (16)$$

- Government spending share:

$$\ln g_t = (1 - \rho_g) \ln g^* + \rho_g \ln g_{t-1} + \epsilon_{g,t}. \quad (17)$$

- Monetary policy shock is serially uncorrelated.
- Define:  $\epsilon_t = [\epsilon_{R,t}, \epsilon_{g,t}, \epsilon_{z,t}]'$



## A Prototypical DSGE Model

- Derive first-order conditions for households' and firms' problems.
- Since non-stationary technology process  $A_t$  induces a stochastic trend in output and consumption, express the model in terms of detrended variables  $c_t = C_t/A_t$  and  $y_t = Y_t/A_t$ .
- Derive the steady state (set the shocks  $\epsilon_t$  equal to zero):
  - Assume that the steady state inflation  $\pi$  equals the target rate  $\pi^*$ .
  - One can show

$$r = \frac{\gamma}{\beta}, \quad R = r\pi^*, \quad c = (1 - \nu)^{1/\tau}, \quad \text{and} \quad y = g(1 - \nu)^{1/\tau}. \quad (18)$$

- Let  $\hat{x}_t = \ln(x_t/x)$  denote the percentage deviation of  $x_t$  from its steady state  $x$ .

## A Prototypical DSGE Model

- The DSGE model can be expressed as

$$1 = \beta \mathbb{E}_t \left[ e^{-\tau \hat{c}_{t+1} + \tau \hat{c}_t + \hat{R}_t - \hat{z}_{t+1} - \hat{\pi}_{t+1}} \right] \quad (19)$$

$$\begin{aligned} \frac{1-\nu}{\nu \varphi \pi^2} (e^{\tau \hat{c}_t} - 1) &= (e^{\hat{\pi}_t} - 1) \left[ \left(1 - \frac{1}{2\nu}\right) e^{\hat{\pi}_t} + \frac{1}{2\nu} \right] \\ &\quad - \beta \mathbb{E}_t \left[ (e^{\hat{\pi}_{t+1}} - 1) e^{-\tau \hat{c}_{t+1} + \tau \hat{c}_t + \hat{y}_{t+1} - \hat{y}_t + \hat{\pi}_{t+1}} \right] \end{aligned} \quad (20)$$

$$e^{\hat{c}_t - \hat{y}_t} = e^{-\hat{y}_t} - \frac{\varphi \pi^2 g}{2} (e^{\hat{\pi}_t} - 1)^2 \quad (21)$$

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) \psi_1 \hat{\pi}_t + (1 - \rho_R) \psi_2 (\hat{y}_t - \hat{g}_t) + \epsilon_{R,t} \quad (22)$$

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + \epsilon_{g,t} \quad (23)$$

$$\hat{z}_t = \rho_z \hat{z}_{t-1} + \epsilon_{z,t} \quad (24)$$

- For output growth rule specification:

$$\hat{R}_t = \rho_R \hat{R}_{t-1} + (1 - \rho_R) \psi_1 \hat{\pi}_t + (1 - \rho_R) \psi_2 (\Delta \hat{y}_t + \hat{z}_t) + \epsilon_{R,t}. \quad (25)$$

## A Prototypical DSGE Model

- Define

$$s_t = [\hat{y}_t, \hat{c}_t, \hat{\pi}_t, \hat{R}_t, \epsilon_{R,t}, \hat{g}_t, \hat{z}_t]'$$

- The solution of the rational expectations system takes the form

$$s_t = \Phi(s_{t-1}, \epsilon_t; \theta). \tag{26}$$

- We will now relate  $s_t$  to the observables...

## A Prototypical DSGE Model

- Suppose we observe:
  - Output growth, quarter-to-quarter, in percent;
  - Inflation, annualized, in percent;
  - Interest Rates, annualized, in percent;
- Time period  $t$  in model corresponds to 1 quarter.
- Measurement equations

$$YGR_t = \gamma^Q + \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t \tag{27}$$

$$INFL_t = \pi^{(A)} + 4 \times \hat{\pi}_t$$

$$INT_t = \pi^{(A)} + r^{(A)} + 4 \times \gamma^{(Q)} + 4 \times \hat{R}_t.$$

## A Prototypical DSGE Model

- The parameters  $\gamma^Q$ ,  $\pi^{(A)}$ , and  $r^{(A)}$  are related to the steady states of the model economy as follows

$$\gamma = 1 + \frac{\gamma^{(Q)}}{100}, \quad \beta = \frac{1}{1 + r^{(A)}/400}, \quad \pi = 1 + \frac{\pi^{(A)}}{400}.$$

- The structural parameters are collected in the vector  $\theta$ . Let

$$\theta = [\tau, \kappa, \psi_1, \psi_2, \rho_R, \rho_g, \rho_z, r^{(A)}, \pi^{(A)}, \gamma^{(Q)}, \sigma_R, \sigma_g, \sigma_z]'$$

Note: subsequently, we will work with log-linear approximations. Since in the first-order approximation the parameters  $\nu$  and  $\varphi$  are not separately identifiable, we express the model in terms of the slope of the Phillips curve  $\kappa$ .